

HoloFed: Environment-Adaptive Positioning via Multi-band Reconfigurable Holographic Surfaces and Federated Learning

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Abstract

Positioning is an essential service for various applications and is expected to be integrated with existing communication infrastructures in 5G and 6G. Though current Wi-Fi and cellular base stations (BSs) can be used to support this integration, the resulting precision is unsatisfactory due to the lack of precise control of the wireless signals. Recently, BSs adopting reconfigurable holographic surfaces (RHSs) have been advocated for positioning as RHSs' large number of antenna elements enable generation of arbitrary and highly-focused signal beam patterns. However, existing designs face two major challenges: i) RHSs only have limited operating bandwidth, and ii) the positioning methods cannot adapt to the diverse environments encountered in practice. To overcome these challenges, we present HoloFed, a system providing high-precision environment-adaptive user positioning services by exploiting *multi-band* (MB)-RHS and *federated learning* (FL). For improving the positioning performance, a lower bound on the error variance is obtained and utilized for guiding MB-RHS's digital and analog beamforming design. For better adaptability while preserving privacy, an FL framework is proposed for users to collaboratively train a position estimator, where we exploit the transfer learning technique to handle the lack of position labels of the users. Moreover, a scheduling algorithm for the BS to select which users train the position estimator is designed, jointly considering the convergence and efficiency of FL. Our performance evaluation based on simulations confirms that HoloFed achieves a 57% lower positioning error variance compared to a beam-scanning baseline and can effectively adapt to diverse environments.

Index Terms

Positioning, reconfigurable holographic surfaces, beamforming, federated learning.

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I. INTRODUCTION

In 5G and 6G wireless systems, positioning is an essential service fundamental to both user location awareness and improved communication [2], [3], and thus has an ever-expanding range of applications in civil and military scenarios [4]. Among all the available positioning techniques, the satellite-based Global Positioning System (GPS) is the most widely used one and can achieve high precision in ideal outdoor environments. Nevertheless, it has the drawbacks of consuming a lot of energy and frequently losing track when buildings block the satellite signals [5], [6]. The loss of GPS signals can take place in various outdoor, indoor, and underground scenarios, creating many GPS-deprived regions where users receive poor positioning services.

To provide positioning services in GPS-deprived regions, many different GPS-free alternatives have been studied, including video-based [7], radar-based [8], and radio frequency identification (RFID)-based [9] positioning techniques. Though the above-mentioned positioning techniques can achieve high precision, they all require additional infrastructure which can be cost-prohibitive for realizing ubiquitous positioning [6]. To reduce the infrastructure cost, integrated sensing and communications (ISAC) has been proposed as a key enabling technology for 6G, integrating sensing and positioning functions into the existing communication infrastructures, e.g., Wi-Fi and cellular base stations (BSs). Nevertheless, such piggyback positioning systems generally cannot ensure high positioning precision, mainly due to their limited bandwidth and comparatively low number of antenna elements.

Thanks to the recent development of metamaterial-based reconfigurable holographic surfaces (RHSs), one may achieve a cost-efficient increase in communication rate, while potentially enhancing the precision of the piggyback positioning services at the same time. This potential is largely attributed to RHSs' characteristic of comprising a massive number of metamaterial antenna elements (*meta-elements* for short), which are densely arranged and have much smaller spatial spacing than half of their operating wavelength. Such a dense arrangement enables RHSs to synthesize arbitrary wavefronts and beam shapes [10], suggesting their strong capability in manipulating electromagnetic (EM) waves [10]. Utilizing this capability, BSs equipped with an RHS can focus transmitted signals into sharp beams to enhance the signal-to-noise ratio (SNR) of users and to probe the region of interest with high resolution and precision.

A few RHS-based systems have been proposed for positioning in the literature [10], [11]: an RHS-based ISAC system is proposed to generate signal beams for both sensing and communi-

cation with high gains in [10], and an RHS is leveraged for target detection with high accuracy yet at low power and cost in [11]. In this context, it is also worthwhile to mention positioning methods exploiting reconfigurable intelligent surfaces (RISs) [2], [12]–[17] due to their intrinsic similarity. Although RISs differ from RHSs in i) the signal feeding scheme (RISs’ over-the-air propagation as opposed to RHSs’ on-board propagation) and thus ii) take up a larger space, the positioning methods for RIS- and RHS-based systems are largely comparable since they both leverage massive numbers of meta-elements for analog beamforming. In [12], the authors utilize an RIS to generate distinguishable signals at different positions and employ a positioning method based on maximum likelihood estimation (MLE). The authors of [13], [14] also employ MLE-based positioning methods, and they optimize the beamforming of the RIS by minimizing the Cramer-Rao lower bound (CRLB) on the positioning error. The beamforming optimization problem is then extended to scenarios involving multiple RISs and obstacles in [15]. In addition to the MLE-based methods, the authors of [2], [16] propose positioning methods based on estimating the time differences among the signals arriving from an RIS. Moreover, the authors of [17] exploit supervised learning to determine the RIS beamformer and the position of user.

The positioning systems and methods discussed above, albeit promising, still have deficiencies in their hardware and software designs, preventing them from being deployed to diverse practical environments. *Firstly*, in the hardware domain, most existing works have considered positioning using signals with rather limited bandwidth, resulting in deficient range resolution and low adaptivity to the frequency selectivity of diverse environments caused by multipath fading. The limited operating bandwidth of existing designs is partially attributed to the physical implementation of meta-elements, which is intrinsically highly frequency selective [18], [19], leading to a severe beam-squinting problem for signals with wide and ultra-wide bandwidth [20]. This means that a single configuration of meta-elements cannot provide the desired beam patterns over a large bandwidth simultaneously as the meta-elements’ signal radiation coefficients vary largely across the band in terms of their phases and amplitudes. In this regard, multi-band (MB) transmission [21], [22] is a promising alternative to ultra-wideband transmission. [The feasibility of MB-RHSs has been verified in \[18\], where an MB-RHS capable of operating in bands at 9.5, 10, 10.5, and 11 GHz is realized. Moreover, in \[23\], \[24\], the authors prototyped RISs employing meta-elements capable of operating in two bands.](#) Exploiting MB transmissions, RHS-based BSs can leverage a larger bandwidth while concurrently realizing appropriate beam patterns.

Secondly, in the software domain, most positioning methods rely on raw received signals or

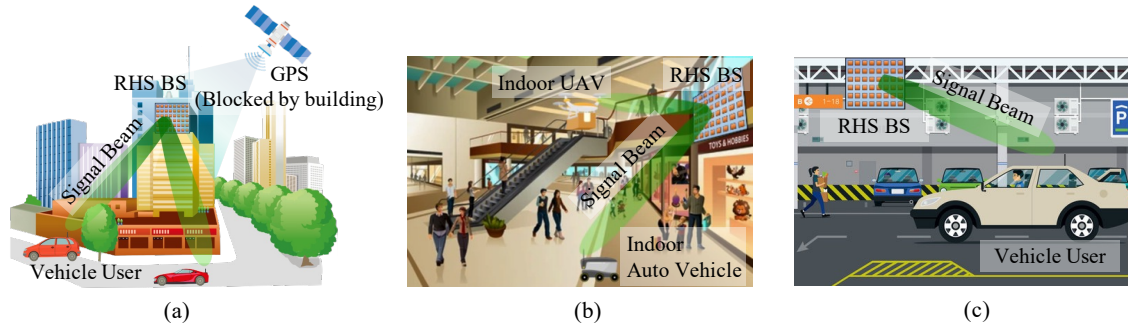


Fig. 1. Application scenarios of HoloFed. (a) Outdoor vehicle positioning for GPS-deprived regions. (b) Indoor autonomous vehicle (e.g., unmanned aerial vehicles (UAVs) and cleaning robots) positioning for large malls. (c) Underground vehicle positioning for garage parking.

extracted features such as time-of-arrival (ToA) and angle-of-arrival (AoA). Such methods lack environmental adaptivity as they cannot fully exploit the environment-specific features contained in the received signals for precision maximization [25]. Although a few recent proposals have started to leverage deep learning techniques for automatic and environment-specific feature selection and extraction [17], [26], they need massive data on received signals and position labels for training. To obtain the required data and labels, *crowd sensing techniques*, where a crowd of users gather the data collaboratively [3], [27], can be potentially exploited. Nevertheless, it is nontrivial to provide effective incentives for users to disclose their position labels since these labels can indicate personal interests and hence potentially compromise user privacy [28], [29].

To achieve high position precision and environmental adaptivity while preserving user privacy, we propose *HoloFed*, an ISAC system with positioning capability specifically targeting a wide range of outdoor, indoor, and underground GPS-deprived scenarios, as shown in Figs. 1(a)-(c). HoloFed exploits MB-RHS and federated learning (FL) to provide the environmental adaptivity in the hardware and software domains, respectively. We propose a positioning protocol under the FL framework for HoloFed, allowing the users to collaboratively adapt HoloFed to diverse environments while preserving the privacy of their own position labels. With the proposed protocol, the BS first transmits signals to the users, utilizing the digital and analog (DA) beamforming capability of MB-RHS. Each user then employs a *position estimator* function distributed by the BS to process the received signals and estimate its position. Furthermore, each user trains the position estimator with its local data, and the BS schedules the users to send the trained position estimator in the uplink to perform global updates.

To optimize the performance of HoloFed, we derive a lower bound on the mean squared error (MSE) of positioning considering the influence of MB multipath fading; this bound is then

exploited to optimize the DA beamforming. Besides, to facilitate FL in practice where users have few position labels, we exploit the transfer learning technique to handle the insufficient training data. Moreover, a user scheduling algorithm is designed for FL, jointly considering the convergence and efficiency. The main contributions of this paper can be summarized as follows:

- We propose the first positioning system assisted by both MB-RHS and FL, delivering low positioning error and high environmental adaptivity without compromising the users' privacy.
- We derive a lower bound on HoloFed's positioning error variance and utilize it for the optimization of DA beamforming of MB-RHSs. Besides, in FL, we handle the lack of users' position labels by exploiting the transfer learning technique and design a user scheduling algorithm to optimize the convergence and efficiency of the training.
- We verify the effectiveness of HoloFed through extensive simulations. Our results confirm that the proposed algorithm is more efficient compared to two benchmark algorithms for beamforming optimization and user scheduling. The results also demonstrate that HoloFed can effectively adapt to diverse environments and achieve low positioning errors.

Compared to its conference version [1], this paper proposes the application of FL for achieving privacy-preserving environmental adaptivity. Furthermore, it provides new optimization algorithms, which enhance the efficiency of the DA beamforming optimization by proximal stochastic descent, handle the insufficiency of users' local data by transfer learning, and improve the efficiency of user scheduling in FL based on an new analytical result of the convergency rate. The remainder of this paper is organized as follows: In Sec. II, the system model for HoloFed is established. Then, for positioning error minimization, an optimization problem is formulated in Sec. III. In Sec. IV, we propose an efficient algorithm for HoloFed to solve the formulated problem. In Sec. V, simulation results are provided, and conclusions are drawn in Sec. VI.

Notations: $\overline{(\cdot)}$, $(\cdot)^\top$, $(\cdot)^H$, and $(\cdot)^{-1}$ are the conjugate, transpose, Hermitian transpose, and inverse operators, respectively. \odot and \otimes denote the Hadamard and Kronecker products, respectively. $\mathbb{R}^{M \times N}$ and $\mathbb{C}^{M \times N}$ denote the sets of real and complex $M \times N$ matrices, respectively. $\mathbf{1}_M$ represents the M -dimensional all-ones column vector, and $\mathbf{1}_{M \times N}$ is the $M \times N$ all-ones matrix. Functions $\text{tr}(\cdot)$ and $\text{diag}(\cdot)$ return the trace and the main diagonal vector of a matrix, respectively. Function $\mathbb{E}_{\mathbf{x} \sim \Gamma}(\cdot)$ returns the expectation of the argument, given variable \mathbf{x} follows distribution Γ . Operators $\|\cdot\|_1$ and $\|\cdot\|_2$ denote the ℓ_1 - and ℓ_2 -norms, respectively. $\nabla_{\mathbf{x}} f$ represents the gradient vector of function f with respect to \mathbf{x} . Symbol \mathbf{i} is the imaginary unit. $[\mathbf{x}]_m$, $[\mathbf{X}]_m$,

and $[\mathbf{X}]_{m,n}$ denote the m -th element of vector \mathbf{x} , the m -th row vector of matrix \mathbf{X} , and the (m,n) -th element of matrix \mathbf{X} , respectively. $\{\mathbf{x}_i\}_i$ is the set of \mathbf{x}_i for all subscript i within its range. $\mathbf{x}^{\circ 2}$ is the element-wise second power of \mathbf{x} . $\Re(\cdot)$ is the real part of the argument.

II. SYSTEM MODEL

The proposed HoloFed is an ISAC system with positioning functionality exploiting an MB-RHS and FL. As in [13], we assume the system utilizes the orthogonal frequency division multiplexing (OFDM) waveform, which is typically adopted for ISAC systems due to its high spectral efficiency, robustness against multipath fading, and easy implementation [30]. As shown in Fig. 2, the system comprises a BS equipped with an RHS and U users. Possible users include cars, autonomous vehicles (UAVs and cleaning robots), and mobile phones. The BS provides data and positioning services for the users in a time-division duplex (TDD) manner. In this paper, we focus on developing the positioning function of HoloFed for a 3D region of interest (ROI).

Specifically, in HoloFed, the process for users to obtain their positions is referred to as the *positioning process*. As the users' positions constitute private information, they are not intended to be known by the BS without users' explicit acknowledgement. Thus, the users in HoloFed estimate their positions by themselves, instead of relying on the BS to estimate their positions and then inform them. This self-positioning is done by using a function referred to as *position estimator*. Nevertheless, since it is hard for individual users to determine the exact characteristics of the RHS and the environment, they cannot effectively derive the position estimator. To handle this issue, the position estimator is provided to the users by the BS. Moreover, to make HoloFed environment-adaptive, the users train the position estimator collaboratively. For the positioning process, a *federated positioning protocol* is proposed as detailed in this section.

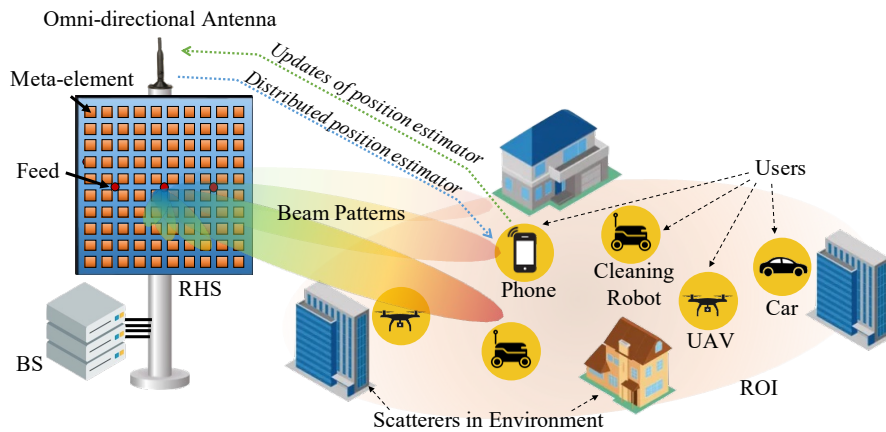


Fig. 2. Illustration of the proposed HoloFed system.

In the following, the components of the system are described in Sec. II-A, the users' received signals are modeled in Sec. II-B, and the federated positioning protocol is provided in Sec. II-C.

A. System Components

The complete HoloFed system comprises an RHS-based BS and multiple users.

1) *RHS-based BS*: The BS is equipped with an RHS, which is a rectangular planar antenna array composed of N_E reconfigurable meta-elements and K signal feeds. Each meta-element can be modelled as an electronic controllable micro-antenna element, which takes the input RF signals from the feeds and radiates the signals into space. Besides, by electronically configuring the different *states* of the meta-element, its signal *radiation coefficient* can be controlled, which determines the ratio between the signals emitted by and fed into it. Based on [10], [31], the radiation coefficient is assumed to be a real number from set¹ $\mathcal{R} = [0, 1]$.

For positioning, the BS transmits OFDM signals in each of N_B bands, where the spectral interval between different bands is assumed to be large to exploit spectrum diversity. Each band has N_{SB} sub-bands, and the bandwidth of each sub-band is W . We assume that the radiation coefficient of a meta-element in a certain state is constant within the sub-bands of the same band as in [10]. Nevertheless, due to the large spectral interval between different bands and the frequency selectivity of the meta-elements, the radiation coefficient of a meta-element in a certain state varies across different bands [18], [19].

Besides, as for communication, since we focus on the positioning function of HoloFed, we assume a simple setting where the BS employs an omnidirectional antenna and communicates with the users by single-band OFDM². With the omnidirectional antenna, the BS broadcasts beaconing frames, receives data from the users, and sends control signals to the users.

2) *Users*: The users are assumed to have omnidirectional single Tx and Rx antennas, and they can communicate with the BS in each of the N_B bands. Nevertheless, due to bandwidth limitation, a user can only transmit or receive signals over one of the N_B bands at a time. Besides, assuming each positioning process has a very short time duration, then the position of a user during a positioning process can be considered fixed and denoted by vector $\mathbf{p} \in \mathcal{P} \subseteq \mathbb{R}^3$ with \mathcal{P} denoting the ROI. Moreover, we assume that the positioning processes take place periodically. In

¹If \mathcal{R} is a discrete set, the method proposed in this paper can still be employed by adding an extra quantization step.

²While the RHS can also be used for communication, this is beyond the scope of this paper. Furthermore, involving it will incur additional complexity for the design of HoloFed's positioning function, and thus it is not considered.

each positioning process, the positions of the users are assumed to be independent and identically distributed random variables, each following a distribution Γ^U , i.e., $\mathbf{p} \sim \Gamma^U$.

B. Received Signal Model

We establish the model for the received signals for the frame transmission from the BS to user n ($n \in \{1, \dots, U\}$), which is also referred to as the *channel model*. Without loss of generality, we index the frame by q , and omit subscript n for the conciseness of presentation. The signals transmitted by the BS and received by the user undergo three stages of propagation, which are referred to as *feed*→*meta-element*, *meta-element radiation*, and *meta-element*→*user*.

1) *Feed*→*meta-element*: Denote the positions of feed k and meta-element m by \mathbf{p}_k^F and \mathbf{p}_m^E ($\forall k \in \{1, \dots, K\}, m \in \{1, \dots, N_E\}$), respectively. Then, based on [10] and [13], for sub-band j of band i and frame q ($\forall i \in \{1, \dots, N_B\}, j \in \{1, \dots, N_{SB}\}, q \in \{1, \dots, F\}$, where F is the number of frames), the incident signals of meta-element m can be expressed as

$$\omega_{i,j,m}^{(q)} = \sum_{k=1}^K s_{i,j,k}^{(q)} \cdot g_{i,j}^F(\boldsymbol{\varphi}_{k,m}^{\text{AoD}}) \cdot \kappa(f_{i,j}, \mathbf{p}_k^F, \mathbf{p}_m^E) \cdot g_{i,j}^E(\boldsymbol{\varphi}_{k,m}^{\text{AoA}}), \quad (1)$$

where $s_{i,j,k}^{(q)}$ denotes the *digital symbol* transmitted by the BS to the RHS via feed k , $g_{i,j}^F(\cdot)$ and $g_{i,j}^E(\cdot)$ represent the *gain patterns* of the feed and the meta-element, respectively, $\boldsymbol{\varphi}_{k,m}^{\text{AoD}}, \boldsymbol{\varphi}_{k,m}^{\text{AoA}} \in \mathbb{R}^{2 \times 1}$ are the angle of departure (AoD) and the angle of arrival (AoA) between feed k and meta-element m , respectively, $f_{i,j}$ is the center frequency of sub-band j of band i , and $\kappa(f_{i,j}, \mathbf{p}_k^F, \mathbf{p}_m^E)$ represents the gain of the on-board propagation from \mathbf{p}_k^F to \mathbf{p}_m^E at frequency $f_{i,j}$ (see [10]):

$$\kappa(f_{i,j}, \mathbf{p}_k^F, \mathbf{p}_m^E) = \exp\left(-\mathbf{i} \cdot \frac{2\pi n_r f_{i,j}}{v_0} \cdot \|\mathbf{p}_m^E - \mathbf{p}_k^F\|_2\right). \quad (2)$$

Here, v_0 is the speed of light and n_r is the refractive index of the RHS board. Moreover, in each frame q , the K digital symbols transmitted by the BS in each sub-band j are bounded by a power constraint, i.e., $\sum_{k=1}^K \|s_{i,j,k}^{(q)}\|_2^2 = P_{\max}$, where P_{\max} is the maximum transmit power³.

2) *Meta-element radiation*: Then, for frame q and band i , the incident signals to each meta-element m are influenced by its radiation coefficient denoted by $c_{i,m}^{(q)}$, which is assumed to be constant for the sub-bands of band i as described in Sec. II-A1. Thus, in sub-band j of band i , the radiated signals of meta-element m in frame q can be expressed as

$$\tau_{i,j,m}^{(q)} = c_{i,m}^{(q)} \cdot \omega_{i,j,m}^{(q)}. \quad (3)$$

³Assuming P_{\max} is fully utilized maximizes the received SNR of the users, which helps to minimize the positioning errors.

3) *Meta-element*→*user*: The radiated signals are then received by the users. For sub-band j of band i , the received signal of the user in frame q can be expressed as

$$\mathbf{y}_{i,j}^{(q)} = \sum_{m=1}^{N_E} (h_{i,j,m}^{\text{LoS}} + h_{i,j,m}^{\text{MP},(q)}) \cdot \tau_{i,j,m}^{(q)} + e_{i,j}^{(q)}, \quad (4)$$

where $e_{i,j}^{(q)} \sim \mathcal{CN}(0, \sigma^2)$ is the thermal noise following the complex Gaussian distribution with variance σ^2 , and $h_{i,j,m}^{\text{LoS}}$ and $h_{i,j,m}^{\text{MP},(q)}$ are the line-of-sight (LoS) and multipath gains, respectively.

Denoting the power spectral density of the noise by P_N , the variance can be expressed as $\sigma^2 = P_N W$. We note that, in (4), the BS and the users are assumed to be fully synchronized as in [13]. Then, based on the signal propagation model in [2] and [32], $h_{i,j,m}^{\text{LoS}}$ can be modelled as

$$h_{i,j,m}^{\text{LoS}} = \frac{v_0 \cdot g_{i,j}^{\text{E}}(\boldsymbol{\theta}_m^{\text{AoD}}) \cdot g_{i,j}^{\text{U}}}{4\pi f_{i,j} \cdot \|\mathbf{p} - \mathbf{p}_m^{\text{E}}\|_2} \cdot \exp\left(-\mathbf{i} \frac{2\pi f_{i,j}}{v_0} \cdot \|\mathbf{p} - \mathbf{p}_m^{\text{E}}\|_2\right). \quad (5)$$

Here, $g_{i,j}^{\text{U}}$ denotes the gain of the user's Rx antenna for sub-band j of band i . The Rx antennas of the U users are assumed to have the identical gains. Besides, $\boldsymbol{\theta}_m^{\text{AoD}} \in \mathbb{R}^{2 \times 1}$ is the AoD of the signals from meta-element m to the user.

Based on [33], [34], we model multipath gains as complex Gaussian random variables satisfying wide-sense stationary condition. Defining $\mathbf{h}_{i,j}^{\text{MP},(q)} = (h_{i,j,1}^{\text{MP},(q)}, \dots, h_{i,j,N_E}^{\text{MP},(q)})^\top$, based on [35], $\mathbf{h}_{i,j}^{\text{MP},(q)} \sim \mathcal{CN}(\mathbf{0}, \mathbf{V}_i)$, where covariance matrix $\mathbf{V}_i \in \mathbb{C}^{N_E \times N_E}$ can be derived from the expectation of the outer product of the RHS's array response $\boldsymbol{\alpha}_i(\boldsymbol{\theta}) \in \mathbb{C}^{N_E}$ over the angular domain⁴, i.e.,

$$\mathbf{V}_i = \mathbb{E}(\boldsymbol{\alpha}_i(\boldsymbol{\theta})\boldsymbol{\alpha}_i(\boldsymbol{\theta})^{\text{H}}) = \oint \boldsymbol{\alpha}_i(\boldsymbol{\theta})\boldsymbol{\alpha}_i(\boldsymbol{\theta})^{\text{H}} P_{\text{pap},i}(\boldsymbol{\theta}) \text{d}\boldsymbol{\theta}. \quad (6)$$

Here, $[\boldsymbol{\alpha}_i(\boldsymbol{\theta})]_m = \exp(\mathbf{i} \frac{2\pi f_i}{v_0} (\mathbf{p}_m^{\text{E}} - \mathbf{p}_1^{\text{E}}) \cdot \hat{\mathbf{n}}(\boldsymbol{\theta})) \cdot g_i^{\text{E}}(\boldsymbol{\theta})$, where f_i is the center frequency of band i , $\hat{\mathbf{n}}(\boldsymbol{\theta})$ is the unit normal vector for $\boldsymbol{\theta}$, and $g_i^{\text{E}}(\cdot)$ is the gain pattern of a meta-element at f_i . Besides, $P_{\text{pap},i}(\boldsymbol{\theta})$ is the *power-angle profile* [34], which accounts for the angular distribution of multipath gains. We note that \mathbf{V}_i can also account for the passive interference among users, i.e., the interference caused to a given user by signals passively scattered by the bodies of other users; because the scattering paths can be modelled as random multipath components.

Based on [35], we model the covariance matrix between the multipath gain vectors for different frames and sub-bands of band i as

$$\mathbb{E}\left(\mathbf{h}_{i,j_1}^{\text{MP},(q_1)} (\mathbf{h}_{i,j_2}^{\text{MP},(q_2)})^{\text{H}}\right) = \rho_{\text{f},i}(j_1, j_2) \cdot \rho_{\text{t},i}(q_1, q_2) \cdot \mathbf{V}_i, \quad \forall j_1, j_2 \in \{1, \dots, N_{\text{SB}}\} \text{ and } q_1, q_2 \in \{1, \dots, F\},$$

⁴Here, we assume that \mathbf{V}_i only depends on the multi-band index i since the multipath gains satisfy the wide-sense stationary condition, and the sub-band frequencies are close to the center frequency of band i .

where $\rho_{f,i}(j_1, j_2)$ and $\rho_{t,i}(q_1, q_2)$ denote the *coherence coefficients* of sub-bands j_1 and j_2 and frames q_1 and q_2 , respectively. Based on [35], they can be expressed as follows:

$$\rho_{f,i}(j_1, j_2) = \frac{1}{1 + \mathfrak{i}2\pi\sigma_{\text{rms},i}(f_{i,j_1} - f_{i,j_2})}, \quad \rho_{t,i}(q_1, q_2) = J_0(2\pi f_{\text{D},i}\Delta_t \cdot (q_1 - q_2)), \quad (7)$$

where Δ_t denotes the duration of a frame, $\sigma_{\text{rms},i}$ denotes the *root mean square (RMS) power delay spread* of band i , $J_0(\cdot)$ is the *zeroth-order Bessel function of the first kind*, and $f_{\text{D},i} = v_{\text{max}}f_i/v_0$ is the maximum Doppler frequency with v_{max} being the users' maximum speed.

Moreover, as the spectral intervals between different OFDM bands are large, the multipath gain vectors of different bands are assumed to be not correlated, i.e.,

$$\mathbb{E} \left(\mathbf{h}_{i_1, j_1}^{\text{MP}, (q_1)} (\mathbf{h}_{i_2, j_2}^{\text{MP}, (q_2)})^{\text{H}} \right) = \mathbf{0}, \quad \forall i_1 \neq i_2. \quad (8)$$

In summary, for the F transmitted frames, we can arrange the digital symbols in (1) transmitted in sub-band j of band i in a matrix $\mathbf{S}_{i,j} \in \mathbb{C}^{F \times K}$ with $[\mathbf{S}_{i,j}]_{q,k} = s_{i,j,k}^{(q)}$ and arrange the radiation coefficients in (3) for band i in a matrix $\mathbf{C}_i \in \mathbb{C}^{F \times N_{\text{E}}}$ with $[\mathbf{C}_i]_{q,m} = c_{i,m}^{(q)}$. Since $\{\mathbf{S}_{i,j}\}_j$ and \mathbf{C}_i control the DA beamforming, we refer to them as the *DA beamforming configuration for band i* . Based on (1), (3), and (4), the received signals of a user for band i are collected in vector $\mathbf{y}_i(\cdot) \in \mathbb{R}^{FN_{\text{SB}} \times 1}$, which is a function of \mathbf{p} , $\{\mathbf{S}_{i,j}\}_j$, and \mathbf{C}_i , and can be expressed as

$$\mathbf{y}_i(\mathbf{p}; \{\mathbf{S}_{i,j}\}_j, \mathbf{C}_i) = \text{diag} \left((\mathbf{H}_i^{\text{LoS}} \otimes \mathbf{1}_F + \mathbf{H}_i^{\text{MP}}) \mathbf{T}_i^{\text{T}} \right) + \mathbf{e}_i. \quad (9)$$

Here, $\mathbf{e}_i \sim \mathcal{CN}(0, \sigma^2 \mathbf{I}_{FN_{\text{SB}}})$ is the noise vector, and the elements of the matrices appearing in (9) can be expressed as follows ($\forall j \in \{1, \dots, N_{\text{SB}}\}, k \in \{1, \dots, K\}, m \in \{1, \dots, N_{\text{E}}\}, q \in \{1, \dots, F\}$):

$$\begin{aligned} [\mathbf{H}_i^{\text{LoS}}]_{j,m} &= h_{i,j,m}^{\text{LoS}}, \quad [\mathbf{H}_i^{\text{MP}}]_{(q-1)N_{\text{SB}}+j,m} = h_{i,j,m}^{\text{MP}, (q)}, \\ [\mathbf{T}_i]_j &= \mathbf{C}_i \odot (\mathbf{S}_{i,j} \mathbf{B}_{i,j}), \quad [\mathbf{B}_{i,j}]_{k,m} = g_{i,j}^{\text{F}}(\varphi_{k,m}^{\text{AoD}}) \cdot g_{i,j}^{\text{E}}(\varphi_{k,m}^{\text{AoA}}) \cdot \kappa(f_{i,j}, \mathbf{p}_k^{\text{F}}, \mathbf{p}_m^{\text{E}}). \end{aligned} \quad (10)$$

Remark 1: In the established model, several parameters are highly sensitive to the hardware implementation and environment and hard to obtain precisely. For instance, the actual gain pattern of the meta-element, i.e., $g_{i,j}^{\text{E}}(\boldsymbol{\theta}_m^{\text{AoD}})$, generally differs from the theoretical model due to unexpected imperfections in the implementation. Besides, the power-angle profile, i.e., $P_{\text{pap},i}(\boldsymbol{\theta})$, is hard to obtain due to the complex influence of signal scatterers in diverse ROIs. We refer to these parameters as the *environmental characteristics*. HoloFed achieves adaptivity to the environmental characteristics via a federated positioning protocol, which is introduced next.

C. Federated Positioning Protocol

When a user needs to estimate its position, it requests the BS to conduct a positioning

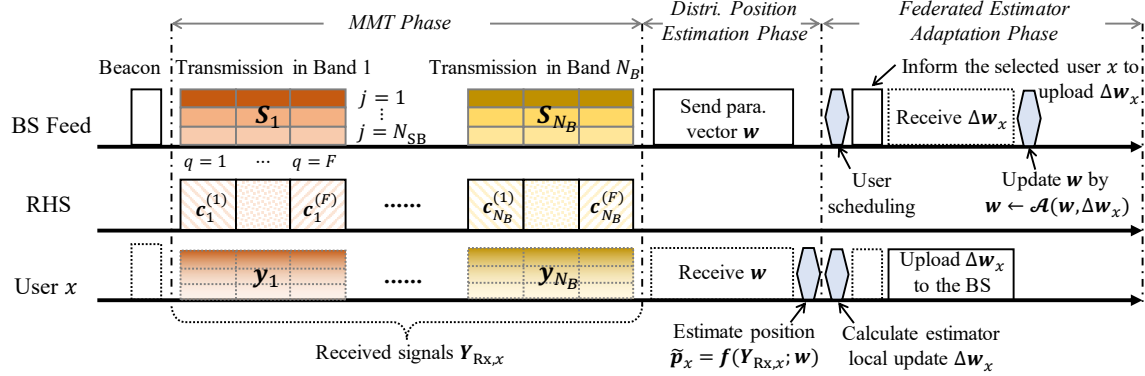


Fig. 3. Federated positioning protocol for HoloFed. The index of the positioning process, t , is omitted to facilitate presentation.

process. After receiving the request, the BS broadcasts a beaconing frame informing the users of the beginning of the positioning process. To coordinate the positioning process and enable HoloFed to adapt to diverse environments, we propose a federated positioning protocol. As shown in Fig. 3, each positioning process is comprised of three phases, i.e., a *MB multi-pattern transmission (MMT) phase*, a *distributed position estimation phase*, and a *federated adaptation phase*. Without loss of generality, we index the positioning process by $t = 0, \dots, T$, and describe the t -th positioning process in the following.

1) *MMT Phase*: In this phase, the BS generates multiple beam patterns in each band by using the RHS, providing users with distinct received signals for position estimation. Since the BS does not know the environmental characteristics or the users' positions, it does not distinguish between different positioning processes, and thus the generated beam patterns are independent of t . Besides, as described in Sec. II-A1, the radiation coefficient of a meta-element in a certain state varies across different bands. Thus, a state configuration that creates a desired beamforming pattern in one band may lead to undesired beam patterns in other bands. Therefore, to design favorable beam patterns in all bands, we assume that the OFDM transmissions in the N_B bands are performed sequentially, allowing the states of the meta-elements to be configured independently in each band. This approach also reduces the hardware requirements for the BS and the users as their RF chains do not have to support ultra-wideband signal transmission and reception⁵.

In each band i , the BS transmits F frames as shown in Fig. 3. Then, for positioning process t , the received signals of user n in all the N_B bands are arranged in matrix $\mathbf{Y}_{\text{Rx},n}^{(t)} \in \mathbb{R}^{N_B \times FN_{\text{SB}}}$, whose i -th row is $[\mathbf{Y}_{\text{Rx},n}^{(t)}]_i = \mathbf{y}_i(\mathbf{p}_n^{(t)}; \{\mathbf{S}_{i,j}\}_j, \mathbf{C}_i)$ based on (9), with $\mathbf{p}_n^{(t)}$ denoting user n 's

⁵If the RHS can independently control the beam patterns in multiple bands, HoloFed can be modified to account for parallel transmissions in these bands, assuming the hardware of both the BS and users is capable of supporting it.

position. To facilitate the presentation, we refer to $\{\mathbf{S}_{i,j}\}_j$ and \mathbf{C}_i for all the N_B bands collectively as the *DA beamforming configuration*, denoted by $\{\mathbf{S}_{i,j}\}_{i,j}$ and $\{\mathbf{C}_i\}_i$, which has an impact on HoloFed's positioning precision since it determines the beam patterns probing the ROI. Based on Fig. 3, the MMT phase has linear time complexity with respect to (w.r.t.) the number of bands and the number of frames transmitted in each band. Besides, since the BS broadcasts the frames to all users at the same time, the MMT phase has constant time complexity w.r.t. the number of users. Consequently, its time complexity is given by $\mathcal{O}(N_B F)$.

2) *Distributed Position Estimation Phase*: In this phase, the BS first distributes the position estimator to the users through downlink beacon transmission. The position estimator is modeled as a multi-layer perceptron (MLP), which is a universal function approximator with high generalization capability [36]. The MLP can be interpreted as a parameterized function with parameter vector $\mathbf{w} \in \mathbb{R}^{N_{\text{para}}}$, where \mathbf{w} is comprised of the N_{para} connection weights and biases of the MLP. Specifically, in positioning process t , the distributed position estimator can be denoted by $\mathbf{f}(\cdot; \mathbf{w}^{(t)}) : \mathbf{Y}_{\text{Rx},n}^{(t)} \rightarrow \tilde{\mathbf{p}}_n^{(t)}$ with $\tilde{\mathbf{p}}_n^{(t)} \in \mathbb{R}^3$ denoting the estimated position of user n .

3) *Federated Adaptation Phase*: Due to the unknown environmental characteristics, the BS cannot effectively determine the position estimator by itself. Thus, in this phase, the users help the BS to adapt the position estimator to the environment by using their local datasets. Here, the local dataset of a user contains the received signal matrices and their corresponding position labels. To enable this adaptation while protecting the privacy of the users' position labels, the *FL framework* is employed, where the position estimator is trained in a distributed manner with no position labels sent to the BS. Specifically, each user first calculates the gradient of the positioning error for its local dataset w.r.t. $\mathbf{w}^{(t)}$. Then, the BS schedules users to upload their gradients, and updates the position estimator based on the received gradients.

More specifically, denote the local dataset gathered by user n by \mathcal{D}_n , and assume that each user has obtained the data-label pairs in \mathcal{D}_n , i.e., $(\mathbf{Y}_{\text{Rx}}, \mathbf{p}) \in \mathcal{D}_n$, when it was near a few *anchors* in the ROI⁶. Then, based on the error measure commonly used for positioning systems, e.g., [39], we assume that user n evaluates its positioning error by the MSE loss, i.e.,

$$\hat{\mathcal{L}}_n(\mathbf{w}^{(t)}) = \sum_{(\mathbf{Y}_{\text{Rx}}, \mathbf{p}) \in \mathcal{D}_n} \|\mathbf{p} - \mathbf{f}(\mathbf{Y}_{\text{Rx}}; \mathbf{w}^{(t)})\|_2^2. \quad (11)$$

⁶Here, an anchor refers to a location where the users can obtain their position labels based on short-range positioning techniques. Such short-range positioning techniques can be readily supported by near-field communication (NFC) of the users with the existing Internet of Things (IoT) infrastructure [37], [38].

Based on (11), the gradient of user n 's local loss can be calculated as $\mathbf{g}_n^{(t)} = \nabla_{\mathbf{w}} \hat{\mathcal{L}}_n(\mathbf{w}^{(t)})$, which is referred to as its *local gradient*. Exploiting the local gradient of the positioning loss w.r.t. the position labels collected near a small number of anchors, HoloFed adapts its position estimator to the actual deployment environment for achieving large-range positioning with high precision.

Moreover, to prevent local gradients from compromising position privacy, the *differentially private (DP) training* mechanism is employed in FL, i.e., noises are added to the local gradients. For user n ($\forall n \in \{1, \dots, U\}$), the noise term added to $\mathbf{g}_n^{(t)}$ is denoted by $\boldsymbol{\varsigma}_n^{(t)} \in \mathbb{R}^{N_{\text{para}}}$ and follows Gaussian distribution $\mathcal{N}(\mathbf{0}, \sigma_{\text{dp},n}^2 \mathbf{I})$. Based on [40], variance $\sigma_{\text{dp},n}^2$ can be calculated as

$$\sigma_{\text{dp},n}^2 = \frac{L^2}{\epsilon_{\text{dp},n}^2} 2 \log(1.25/\delta_{\text{dp}}), \quad (12)$$

where $\epsilon_{\text{dp},n}$ represents the *privacy leakage bound* of user n in terms of differential privacy, L denotes the Lipschitz constant of the local gradient which can be enforced by having each user rescale its local gradient to L in terms of ℓ_2 -norm, and $\delta_{\text{dp}} \ll 1$ is a small constant ensuring $\sigma_{\text{dp},n}^2$ to be finite by allowing a violation probability of the privacy leakage bound. Consequently, the *local update* that user n prepares to upload can be expressed as $\Delta \mathbf{w}_n^{(t)} = -\mathbf{g}_n^{(t)} + \boldsymbol{\varsigma}_n^{(t)}$.

Furthermore, as HoloFed also needs to provide communication services, we assume that in each positioning process, only one user is selected to upload its local update over a single band⁷, so that the occupation of the time-spectrum resources for FL is minimized. For the t -th positioning process, denote the probability of selecting each user for uploading by *scheduling probability vector* $\boldsymbol{\xi}^{(t)} = (\xi_1^{(t)}, \dots, \xi_U^{(t)})$. The update of the parameter vector can be expressed as

$$\mathbf{w}^{(t+1)} = \mathcal{A}^{(t)}(\mathbf{w}^{(t)}, \Delta \mathbf{w}_x^{(t)}) \Big|_{x \sim \mathcal{M}(\boldsymbol{\xi}^{(t)})}, \quad (13)$$

where x is the index of the selected user, $\mathcal{A}^{(t)}(\cdot)$ denotes the *adaptation function* used by the BS to update $\mathbf{w}^{(t)}$ based on $\Delta \mathbf{w}_x^{(t)}$, and $\mathcal{M}(\boldsymbol{\xi}^{(t)})$ denotes the *multinomial distribution* given $\boldsymbol{\xi}^{(t)}$.

Remark 2: In HoloFed, users do not suffer from active user interference caused by signal transmissions of other users because they only receive signals in the MMT phase and are scheduled to transmit their local updates one at a time in the federated adaptation phase.

III. PROBLEM FORMULATION FOR POSITIONING ERROR MINIMIZATION

We formulate an optimization problem for HoloFed, targeting the minimization of the average MSE of positioning experienced by the users over the ROI after adaptation. The degrees of

⁷The band used for uploading can be selected by the user for rate maximization. Even multiple bands can be used if the user and the BS can support it. The proposed algorithm can be modified to accommodate such cases, as described in Sec. IV-C.

freedom for optimization include the DA beamforming configuration, i.e., $\{\mathbf{S}_{i,j}\}_{i,j}$ and $\{\mathbf{C}_i\}_i$, the initial parameter vector of the position estimator, $\mathbf{w}^{(0)}$, and the sets of adaptation functions and scheduling probability vectors for the $t = 0, \dots, T$ positioning processes, i.e., $\{\mathcal{A}^{(t)}\}_t$ and $\{\boldsymbol{\xi}^{(t)}\}_t$. The positioning error minimization problem is formulated as follows:

$$(P1) : \min_{\substack{\{\mathbf{S}_{i,j}\}_{i,j}, \{\mathbf{C}_i\}_i, \\ \mathbf{w}^{(0)}, \{\mathcal{A}^{(t)}\}_t, \{\boldsymbol{\xi}^{(t)}\}_t}} \mathcal{L}(\mathbf{w}^{(T)}) = \sum_{n=1}^U \mathbb{E}_{\mathbf{p}_n \sim \Gamma^U} (\|\mathbf{p}_n - \mathbf{f}(\mathbf{Y}_{\text{Rx},n}; \mathbf{w}^{(T)})\|_2^2), \quad (14a)$$

$$\text{s.t. } [\mathbf{Y}_{\text{Rx},n}]_i = \mathbf{y}_i(\mathbf{p}_n; \{\mathbf{S}_{i,j}\}_j, \mathbf{C}_i), \quad (14b)$$

$$\text{diag}(\mathbf{S}_{i,j} \mathbf{S}_{i,j}^H) = P_{\max} \mathbf{1}_F, \quad (14c)$$

$$[\mathbf{C}_i]_{q,m} \in \mathcal{R}, \quad (14d)$$

$$\Delta \mathbf{w}_n^{(t)} = -\nabla_{\mathbf{w}} \hat{\mathcal{L}}_n(\mathbf{w}^{(t)}) + \boldsymbol{\zeta}_n^{(t)}, \quad (14e)$$

$$\mathbf{w}^{(t+1)} = \mathcal{A}^{(t)}(\mathbf{w}^{(t)}, \Delta \mathbf{w}_x^{(t)}) \Big|_{x \sim \mathcal{M}(\boldsymbol{\xi}^{(t)})}, \quad (14f)$$

$$\mathbf{0} \preceq \boldsymbol{\xi}^{(t)} \preceq \mathbf{1}, \quad \|\boldsymbol{\xi}^{(t)}\|_1 = 1. \quad (14g)$$

$$\forall i \in \{1, \dots, N_B\}, j \in \{1, \dots, N_{\text{SB}}\}, q \in \{1, \dots, F\}, m \in \{1, \dots, N_E\}, \\ n \in \{1, \dots, U\}, t \in \{0, \dots, T\},$$

In (14a), $\mathcal{L}(\mathbf{w}^{(T)})$ denotes the expected MSE of positioning experienced by the U users after the adaptation in the T positioning processes. Constraint (14b) indicates that the relationship between the user position and the received signal matrix follows the channel model established in (9). Constraints (14c) and (14d) indicate that the symbols for digital beamforming satisfy the power constraint, and that the radiation coefficient value of each meta-element belongs to set \mathcal{R} , respectively. Besides, constraints (14e) and (14f) follow from the update of the parameter vector of the position estimator in each positioning process according to the protocol proposed in Sec. II-C3. Moreover, constraint (14g) ensures that $\boldsymbol{\xi}^{(t)}$ is a valid probability vector.

The challenges of solving (P1) comprise the following three aspects: *Firstly*, as described in the remark in Sec. II-B, channel models $\{\mathbf{y}_i(\cdot)\}_i$ in (14b) contain undetermined environmental characteristics, making them hard to evaluate the influence of the DA beamforming configuration on the received signals. This hinders the optimization of $\{\mathbf{S}_{i,j}\}_{i,j}$ and $\{\mathbf{C}_i\}_i$.

Secondly, in (14e), as the local datasets only contain the position labels collected near a few anchors, the loss function of user n , $\hat{\mathcal{L}}_n(\cdot)$, is not identical to the objective function, $\mathcal{L}(\cdot)$. Thus, the update of the parameter vector in (14f) does not necessarily reduce the expected MSE of positioning. To handle this problem, initial parameter vector $\mathbf{w}^{(0)}$ and adaptation function set

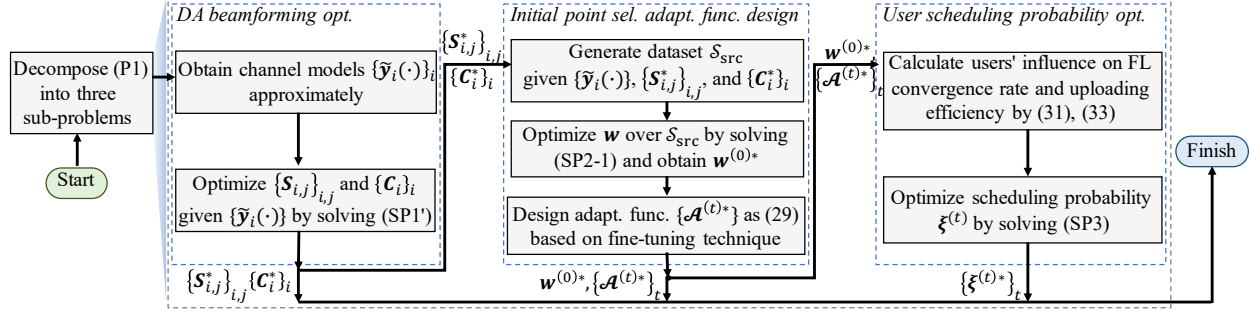


Fig. 4. Flow chart of the proposed positioning error minimization algorithm for HoloFed.

$\{\mathcal{A}^{(t)}\}_t$ need to be properly selected. However, the degrees of freedom of $\mathbf{w}^{(0)}$ and $\{\mathcal{A}^{(t)}\}_t$ are very high due to the large number of parameters in the MLP and the arbitrary forms of the adaptation functions, making the search for $\mathbf{w}^{(0)}$ and $\{\mathcal{A}^{(t)}\}_t$ very challenging.

Thirdly, as the BS does not know the users' local datasets or local gradients when determining the scheduling probability vectors, it is hard for the BS to evaluate the influence of $\xi^{(t)}$ let alone to optimize it. Nevertheless, as $\{\xi^{(t)}\}_t$ in (14f) has an impact on the convergence and efficiency of FL, it has a fundamental influence on the objective function. Therefore, optimizing $\{\xi^{(t)}\}_t$ is both crucial for solving (P1) and very challenging.

In summary, solving (P1) is highly complex and challenging, and is beyond the capabilities of conventional optimization algorithms. Hence, to solve (P1), novel algorithms that can effectively handle the aforementioned three challenges are needed.

IV. POSITIONING ERROR MINIMIZATION ALGORITHM FOR HOLOFED

In this section, we handle (P1) by proposing a *positioning error minimization algorithm for HoloFed*. Specifically, we tackle the three challenges arising when solving (P1) by decomposing the problem into three sub-problems. The three sub-problems are: the *DA beamforming optimization* by solving $\{\mathbf{S}_{i,j}^*\}_{i,j}$ and $\{\mathbf{C}_i^*\}_i$, the *initial point selection and adaptation function design* for $\mathbf{w}^{(0)*}$ and $\{\mathcal{A}^{(t)*}\}_t$, and the *user scheduling probability optimization* by solving $\{\xi^{(t)*}\}_t$. A flow chart of the complete algorithm proposed for solving (P1) is provided in Fig. 4.

A. DA Beamforming Optimization

To overcome the challenge due to undetermined environmental characteristics, we substitute the channel models in (P1), $\{\mathbf{y}_i(\cdot)\}_i$, with deterministic ones denoted by $\{\tilde{\mathbf{y}}_i(\cdot)\}_i$, where the environmental characteristics such as the gain pattern of meta-elements and the power-angle profile are obtained approximately. The reason why we can perform this substitution is that

the federated positioning protocol enables HoloFed to adapt to the environment, allowing the position estimator to adjust to the difference between $\{\mathbf{y}_i(\cdot)\}_i$ and $\{\tilde{\mathbf{y}}_i(\cdot)\}_i$ via FL. Thus, we can solve the DA beamforming optimization problem given $\{\tilde{\mathbf{y}}_i(\cdot)\}_i$ and leave the adaptation to FL. Besides, to facilitate the DA beamforming optimization, we consider that the optimal position estimator for DA beamforming configuration is employed, which is denoted by $\tilde{\mathbf{f}}^*(\cdot)$. Therefore, the optimization of $\{\mathbf{S}_{i,j}\}_{i,j}$ and $\{\mathbf{C}_i\}_i$ in (P1) is converted into a sub-problem:

$$(SP1): \min_{\{\mathbf{S}_{i,j}\}_{i,j}, \{\mathbf{C}_i\}_i} \mathbb{E}_{\mathbf{p} \sim \mathcal{I}^U} \left(\|\mathbf{p} - \tilde{\mathbf{f}}^*(\mathbf{Y}_{\text{Rx}})\|_2^2 \right), \text{ s.t. } [\mathbf{Y}_{\text{Rx}}]_i = \tilde{\mathbf{y}}_i(\mathbf{p}; \{\mathbf{S}_{i,j}\}_j, \mathbf{C}_i), \text{ (14c)-(14d), } \forall i, j.$$

Problem (SP1) is still challenging because: 1) $\tilde{\mathbf{f}}^*(\cdot)$ is undetermined; 2) the noises and multipath gains in the channel models are random variables; 3) the total dimension of $\{\mathbf{S}_{i,j}\}_{i,j}$ and $\{\mathbf{C}_i\}_i$ is very large, namely $N_B F N_{\text{SB}} K + N_B F N_E$, resulting in high computational complexity for evaluating the objective function of (SP1) and its gradient.

To overcome the above challenges, we first convert (SP1) into a CRLB minimization problem, addressing the challenges of the undetermined $\tilde{\mathbf{f}}^*(\cdot)$ and the random noises and multipath gains. Then, to avoid the high computational complexity caused by the large dimension of $\{\mathbf{S}_{i,j}\}_{i,j}$ and $\{\mathbf{C}_i\}_i$, the gradient of the CRLB is obtained in closed form, based on which an efficient stochastic gradient descent algorithm is proposed. The detailed steps for solving (SP1) are described below.

1) *Deriving the CRLB on the MSE of Positioning:* Given $\{\mathbf{S}_{i,j}\}_{i,j}$ and $\{\mathbf{C}_i\}_i$, we first analyze the objective function value of (SP1), which can be obtained by solving optimization problem $\min_{\tilde{\mathbf{f}}(\cdot)} \mathbb{E}_{(\mathbf{Y}_{\text{Rx}}, \mathbf{p})} (\|\mathbf{p} - \tilde{\mathbf{f}}(\mathbf{Y}_{\text{Rx}})\|_2^2)$. Supposing the solution to the optimization problem, $\tilde{\mathbf{f}}^*(\cdot)$, is unbiased, the optimum of the objective in (SP1) can be benchmarked by the average CRLB over the ROI, since the CRLB is a valid lower bound for the variance of any unbiased estimator [41]. Specifically, the CRLB for position \mathbf{p} can be calculated based on the following Proposition 1.

Proposition 1. *Given $\{\mathbf{S}_{i,j}\}_{i,j}$ and $\{\mathbf{C}_i\}_i$, the CRLB for a user at \mathbf{p} can be calculated as*

$$\text{CRLB}(\mathbf{p}) = \sum_{u=1}^3 [\mathbf{I}_{\text{FIM}}^{-1}(\mathbf{p})]_{uu}. \quad (16)$$

Here, $\mathbf{I}_{\text{FIM}}(\mathbf{p})$ is the Fisher information matrix (FIM) of \mathbf{Y}_{Rx} w.r.t. \mathbf{p} , whose (u, v) -th element ($\forall u, v \in \{1, 2, 3\}$) is given by:

$$[\mathbf{I}_{\text{FIM}}(\mathbf{p})]_{u,v} = 2\Re \left(\sum_{i=1}^{N_B} \left(\frac{\partial \hat{\mathbf{y}}_i}{\partial p_u} \right)^H \mathbf{\Lambda}_i^{-1} \left(\frac{\partial \hat{\mathbf{y}}_i}{\partial p_v} \right) \right), \quad (17)$$

where $\hat{\mathbf{y}}_i$ denotes the expectation of $\tilde{\mathbf{y}}_i(\mathbf{p})$, p_u is the u -th element of \mathbf{p} , and $\mathbf{\Lambda}_i \in \mathbb{C}^{F N_{\text{SB}} \times F N_{\text{SB}}}$

is the covariance matrix of $\tilde{\mathbf{y}}_i$. The terms in (17) can be calculated as follows

$$\frac{\partial \hat{\mathbf{y}}_i}{\partial p_u} = \text{diag}((\dot{\mathbf{H}}_{i,u}^{\text{LoS}} \otimes \mathbf{1}_F) \mathbf{T}_i^\top), \quad \mathbf{A}_i = \mathbf{K}_{\text{ft},i} \odot (\mathbf{T}_i \mathbf{V}_i \mathbf{T}_i^H) + \sigma^2 \mathbf{I}, \quad (18)$$

where $\dot{\mathbf{H}}_{i,u}^{\text{LoS}} = \partial \mathbf{H}_i^{\text{LoS}} / \partial p_u$, and $\mathbf{K}_{\text{ft},i} = (\mathbf{K}_{\text{f},i} \otimes \mathbf{1}_{F \times F}) \odot (\mathbf{1}_{N_{\text{SB}} \times N_{\text{SB}}} \otimes \mathbf{K}_{\text{t},i})$ with $[\mathbf{K}_{\text{f},i}]_{j_1, j_2} = \rho_{\text{f},i}(j_1, j_2)$ and $[\mathbf{K}_{\text{t},i}]_{q_1, q_2} = \rho_{\text{t},i}(q_1, q_2)$.

Proof: The FIM in (17) can be obtained by substituting the channel model in (9) into the FIM formula in [42, Eq. (6.55)], and the CRLB can be obtained based on [13, Eq. (27)]. ■

Based on Proposition 1, we use the expected CRLB over the ROI to substitute the objective in (SP1). Moreover, to facilitate the calculation of expectation, we employ the Monte Carlo method to approximate the expected CRLB by the average CRLB for a set of N_{sam} randomly sampled positions following distribution Γ^{U} , which is denoted by \mathcal{S}_{sam} . Thus, (SP1) is converted into the following CRLB minimization problem:

$$(\text{SP1}') : \quad \min_{\{\mathcal{S}_{i,j}\}_{i,j}, \{\mathcal{C}_i\}_i} \sum_{\mathbf{p} \in \mathcal{S}_{\text{sam}}} \frac{\text{CRLB}(\mathbf{p})}{N_{\text{sam}}}, \quad \text{s.t. (14c)-(14d), } \forall i, j. \quad (19)$$

Remark 3: Based on (6) and Proposition 1, the impact of passive user interference on the CRLB can be analyzed: If the magnitudes of all entries of \mathbf{V}_i increase by a factor of X times due to larger passive user interference ($\forall i \in \{1, \dots, N_{\text{B}}\}$), then based on (16)–(18), $\text{CRLB}(\mathbf{p})$ will increase approximately by a factor of X as well.

Remark 4: According to Proposition 1, the benefits of using multiple bands for positioning are two-fold. *Firstly*, the Fisher information from multiple bands adds up, leading to a lower value for the CRLB. *Secondly*, the fact that signals received in different bands are generally less correlated (due to less correlated multipath gains) also contributes to a lower CRLB. This can be shown by deriving the Fisher information of two correlated signals based on [41, Eq. (3.31)].

2) *Solving CRLB Minimization:* In (SP1'), there are a large number of optimization variables in $\{\mathcal{S}_{i,j}\}_{i,j}$ and $\{\mathcal{C}_i\}_i$. Besides, to ensure the objective function in (SP1') approximates the actual expectation of the CRLB over the ROI, N_{sam} also needs to be large. Consequently, evaluating the value and gradients of the objective function is highly computationally complex, which makes traditional optimization algorithms inefficient. To solve (SP1') efficiently, we design a proximal stochastic gradient descent algorithm based on the ProxSARAH algorithm in [43], which is a state-of-the-art proximal stochastic descent algorithm.

Similar to ProxSARAH, our algorithm involves an *inner loop* and an *outer loop*. In the outer loop, the gradient of the objective function w.r.t. the optimization variables is coarsely estimated

for a set of sampling points in the ROI. Then, in the inner loop, the gradient estimate is iteratively refined based on the gradient deviation determined during the update of the optimization variables. This method enhances the precision of gradient estimation for limited sampling points, leading to an accelerated convergence rate [43]. Moreover, to efficiently handle the large number of optimization variables, we optimize $\{\mathbf{S}_{i,j}\}_{i,j}$ and $\{\mathbf{C}_i\}_i$ alternately.

The proposed algorithm is described as follows. In each iteration of the outer loop, the inner loops for the DA beamforming variables are conducted sequentially. In the ℓ_{out} -th iteration ($\ell_{\text{out}} = 1, \dots, N_{\text{out}}$) of the outer loop for instance, a *gradient estimate* is first generated as

$$\mathbf{V}_{\mathbf{S},i,j}^{(0)} = \frac{1}{|\mathcal{B}_{\mathbf{S}}^{(\ell_{\text{out}})}|} \sum_{\mathbf{p} \in \mathcal{B}_{\mathbf{S}}^{(\ell_{\text{out}})}} \nabla_{\mathbf{S}_{i,j}} \text{CRLB}(\mathbf{p}; \mathbf{S}_{i,j}), \quad \forall i, j, \quad (20)$$

where $\mathcal{B}_{\mathbf{S}}^{(\ell_{\text{out}})}$ denotes a randomly selected batch of position samples in \mathcal{S}_{sam} .

Then, the inner loop for digital beamforming starts with $\mathbf{S}_{i,j}^{(0)} = \mathbf{S}_{i,j}$. In the ℓ_{in} -th iteration ($\ell_{\text{in}} = 1, \dots, N_{\text{in}}$) of the inner loop, the digital beamforming variables are updated as

$$\mathbf{S}_{i,j}^{(\ell_{\text{in}})} = (1 - \gamma) \mathbf{S}_{i,j}^{(\ell_{\text{in}}-1)} + \gamma \mathbf{prox}_{\mathbf{S}}(\mathbf{S}_{i,j}^{(\ell_{\text{in}}-1)} - \beta \mathbf{V}_{\mathbf{S},i,j}^{(\ell_{\text{in}}-1)}), \quad \forall i, j, \quad (21)$$

where γ and β are coefficients controlling the step size of the update, and $\mathbf{prox}_{\mathbf{S}}(\cdot)$ denotes the proximal operator for the digital beamforming variables to ensure that constraint (14c) is satisfied. Then, the gradient estimate is iteratively updated in the inner loop as

$$\mathbf{V}_{\mathbf{S},i,j}^{(\ell_{\text{in}})} = \mathbf{V}_{\mathbf{S},i,j}^{(\ell_{\text{in}}-1)} + \sum_{\mathbf{p} \in \mathcal{B}_{\mathbf{S}}^{(\ell_{\text{out}}, \ell_{\text{in}})}} \frac{\nabla_{\mathbf{S}_{i,j}} \text{CRLB}(\mathbf{p}; \mathbf{S}_{i,j}^{(\ell_{\text{in}})}) - \nabla_{\mathbf{S}_{i,j}} \text{CRLB}(\mathbf{p}; \mathbf{S}_{i,j}^{(\ell_{\text{in}}-1)})}{|\mathcal{B}_{\mathbf{S}}^{(\ell_{\text{out}}, \ell_{\text{in}})}|}, \quad \forall i, j, \quad (22)$$

where $\mathcal{B}_{\mathbf{S}}^{(\ell_{\text{out}}, \ell_{\text{in}})}$ is a randomly selected batch of samples in \mathcal{S}_{sam} to estimate the gradient difference. The obtained $\mathbf{V}_{\mathbf{S},i,j}^{(\ell_{\text{in}})}$ is fed into (21) for the next iteration. After N_{in} iterations of the inner loop, the current digital beamforming variables are updated as $\mathbf{S}_{i,j} = \mathbf{S}_{i,j}^{(N_{\text{in}})}, \forall i, j$.

Then, steps similar to (20)-(22) are carried out for the analog beamforming variables, substituting symbols $\mathbf{S}_{i,j}$, $\mathbf{V}_{\mathbf{S},i,j}$, $\mathcal{B}_{\mathbf{S}}$, $\mathbf{prox}_{\mathbf{S}}(\cdot)$, and $\nabla_{\mathbf{S}_{i,j}} \text{CRLB}(\mathbf{p}; \mathbf{S}_{i,j})$ with \mathbf{C}_i , $\mathbf{V}_{\mathbf{C},i}$, $\mathcal{B}_{\mathbf{C}}$, $\mathbf{prox}_{\mathbf{C}}(\cdot)$, and $\nabla_{\mathbf{C}_i} \text{CRLB}(\mathbf{p}; \mathbf{C}_i)$, respectively. Moreover, based on [44], in (21), the proximal operators for the DA beamforming variables can be expressed as

$$[\mathbf{prox}_{\mathbf{S}}(\mathbf{S}_{i,j})]_q = \frac{[\mathbf{S}_{i,j}]_q}{\|[\mathbf{S}_{i,j}]_q\|_2} P_{\max}, \quad [\mathbf{prox}_{\mathbf{C}}(\mathbf{C}_i)]_{q,m} = \min(\max([\mathbf{C}_i]_{q,m}, 0), 1), \quad (23)$$

where $q \in \{1, \dots, F\}$ and $m \in \{1, \dots, N_{\text{E}}\}$. Furthermore, we derive the gradients of the CRLB w.r.t. $\{\mathbf{S}_{i,j}\}_{i,j}$ and $\{\mathbf{C}_i\}_i$ in (20) and (22) in close form in the following proposition.

Proposition 2. *The gradient of the CRLB w.r.t. $\mathbf{S}_{i,j}$ ($\forall i \in \{1, \dots, N_{\text{B}}\}, j \in \{1, \dots, N_{\text{SB}}\}$) at*

Algorithm 1 DA Beamforming Optimization Algorithm

- 1: Sample N_{sam} position samples following Γ^U and obtain \mathcal{S}_{sam} .
 - 2: Set initial $\{\mathbf{S}_{i,j}^{(0)}\}_{i,j} = \{\mathbf{S}_{i,j} | s_{i,j,k}^{(a)} = \sqrt{P_{\text{max}}/K}\}$ and $\{\mathbf{C}_i^{(0)}\}_i$ containing random elements within $[0, 1]$.
 - 3: **for** $\ell_{\text{out}} = 1, \dots, N_{\text{out}}$ **do** # Outer Loop
 - 4: Generate an initial gradient estimate for the CRLB with (20), which is denoted by $\mathbf{V}_{\mathbf{S}_{i,j}}^{(0)}, \forall i, j$.
 - 5: **for** $\ell_{\text{in}} = 1, \dots, N_{\text{in}}$ **do** # Inner Loop
 - 6: Update $\mathbf{S}_{i,j}^{(\ell_{\text{in}})}$ based on (21) and (23) by using gradient estimate $\mathbf{V}_{\mathbf{S}_{i,j}}^{(\ell_{\text{in}}-1)}, \forall i, j$.
 - 7: Update the gradient estimate by (22) with the help of the gradient formulas given in Proposition 2.
 - 8: Conduct steps similar to Steps 4 to 7 for analog beamforming variables $\{\mathbf{C}_i\}_i, \forall i$.
 - 9: Return $\{\mathbf{S}_{i,j}^*\}_{i,j}$ and $\{\mathbf{C}_i^*\}_i$ as the current $\{\mathbf{S}_{i,j}\}_{i,j}$ and $\{\mathbf{C}_i\}_i$.
-

position \mathbf{p} can be calculated by the formulas as follows:

$$\frac{\partial \text{CRLB}(\mathbf{p}; \mathbf{S}_{i,j})}{\partial \mathbf{S}_{i,j}} = -\text{tr}\left(\frac{\partial \mathbf{I}_{\text{FIM}}(\mathbf{p})}{\partial \mathbf{S}_{i,j}} \mathbf{I}_{\text{FIM}}^{-2}(\mathbf{p})\right), \text{ where} \quad (24)$$

$$\left[\frac{\partial \mathbf{I}_{\text{FIM}}(\mathbf{p})}{\partial \mathbf{S}_{i,j}}\right]_{u,v} = 2\Re\left(\mathbf{A}_{i,j,u,v}^s + \mathbf{A}_{i,j,u,v}^s + \mathbf{B}_{i,j,u,v}^s + \mathbf{B}_{i,j,u,v}^s\right), \forall u, v \in \{1, 2, 3\}. \quad (25)$$

Here, notations $\mathbf{A}_{i,j,u,v}^s$ and $\mathbf{B}_{i,j,u,v}^s$ are defined in Appendix A. Besides, the gradient of the CRLB w.r.t. \mathbf{C}_i ($\forall i \in \{1, \dots, N_B\}$) at \mathbf{p} can be calculated in a similar manner as (24), (25) by substituting $\mathbf{A}_{i,j,u,v}^s$, $\mathbf{B}_{i,j,u,v}^s$, and $\mathbf{S}_{i,j}$ with $\mathbf{A}_{i,u,v}^c$, $\mathbf{B}_{i,u,v}^c$, and \mathbf{C}_i , respectively. The complete expressions of $\mathbf{A}_{i,u,v}^c$ and $\mathbf{B}_{i,u,v}^c$ are also given in Appendix A.

Proof: Please refer to Appendix A. ■

In summary, the algorithm for DA beamforming optimization is provided in Algorithm 1. In the following subsections, $\{\mathbf{S}_{i,j}^*\}_{i,j}$ and $\{\mathbf{C}_i^*\}_i$ obtained by Algorithm 1 are employed as default.

B. Initial Point Selection and Adaptation Function Design

Now, we focus on the second challenge of (P1), i.e., the objective function cannot be effectively minimized as the user's local datasets only contain position labels for the areas around a few anchors. To overcome this challenge, a proper adaptation function, $\{\mathcal{A}^{(t)*}\}_t$, and a suitable initial parameter vector of the position estimator, $\mathbf{w}^{(0)*}$, are needed. Nevertheless, due to their high degrees of freedom, they are hard to obtain by traditional optimization algorithms. To tackle this issue effectively, we employ the *transfer learning* technique to obtain $\mathbf{w}^{(0)*}$ and $\{\mathcal{A}^{(t)*}\}_t$.

To begin with, we describe the initial point selection and adaptation function design sub-problem of (P1) in the context of transfer learning as follows. The target environment, where we aim to optimize HoloFed, constitutes *target domain* $\mathcal{D}_{\text{tar}} = \{\Gamma^U, \{\mathbf{y}_i(\cdot)\}_i\}$, which comprises the

distribution of user position Γ^U and the set of exact channel models $\{\mathbf{y}_i(\cdot)\}_i$. Then, the objective of (P1) can be considered as a task on \mathcal{D}_{tar} denoted by $\mathcal{T}(\mathcal{D}_{\text{tar}})$, in which we aim to find the optimal parameter vector for the minimization of the positioning error given Γ^U and $\{\mathbf{y}_i(\cdot)\}_i$.

Due to the undetermined environmental characteristics and the insufficient local datasets of the users, the solution to $\mathcal{T}(\mathcal{D}_{\text{tar}})$ cannot be obtained by conventional optimization techniques. Fortunately, the transfer learning technique provides an effective means to handle $\mathcal{T}(\mathcal{D}_{\text{tar}})$. Specifically, we resort to a domain similar to \mathcal{D}_{tar} , where the task can be efficiently solved. We refer to this domain as the *source domain* and denote it by \mathcal{D}_{src} , and the solution to $\mathcal{T}(\mathcal{D}_{\text{src}})$ is denoted by $\mathbf{w}^{*'}$. To obtain $\mathbf{w}^{*'}$, certain environmental characteristics need to be assumed for \mathcal{D}_{src} , which are generally different from those in \mathcal{D}_{tar} . This results in different joint distributions for the user positions and received signals in the two domains, and hence $\mathbf{w}^{*'}$ is not valid in \mathcal{D}_{tar} . Nevertheless, due to the intrinsic similarities between \mathcal{D}_{tar} and \mathcal{D}_{src} (e.g., the underlying signal propagation models and DA beamforming configurations are identical), transfer learning can be used efficiently to adapt $\mathbf{w}^{*'}$ to \mathcal{D}_{tar} [45]. Therefore, $\mathcal{T}(\mathcal{D}_{\text{tar}})$ can be handled by selecting $\mathbf{w}^{*'}$ as the initial point, i.e., $\mathbf{w}^{(0)}$, and adapting it to \mathcal{D}_{tar} with $\{\mathcal{A}^{(t)}\}_t$, which is designed to minimize the positioning error of $\mathbf{f}(\cdot; \mathbf{w}^{(t)})$ over the users' local datasets. In the following, we describe the detailed procedures for selecting the initial point and designing the adaptation function.

1) *Selection of Initial Point:* We choose the source domain having channel model $\{\tilde{\mathbf{y}}_i(\cdot)\}_i$ in Sec. IV-A, i.e., $\mathcal{D}_{\text{src}} = \{\Gamma^U, \{\tilde{\mathbf{y}}_i(\cdot)\}_i\}$. Then, by using $\{\tilde{\mathbf{y}}_i(\cdot)\}_i$, the BS can generate sufficient received signal matrices with position labels. Denote the generated dataset by $\mathcal{S}_{\text{src}} = \{(\mathbf{Y}_{\text{Rx},\ell}, \mathbf{p}_\ell)\}_\ell$ with $\ell \in \{1, \dots, N_{\text{src}}\}$, where N_{src} is the size of \mathcal{S}_{src} . Then, $\mathcal{T}(\mathcal{D}_{\text{src}})$ can be handled by solving the following optimization problem.

$$(\text{SP2-1}) : \min_{\mathbf{w}'} \frac{1}{N_{\text{src}}} \sum_{(\mathbf{Y}_{\text{Rx},\ell}, \mathbf{p}_\ell) \in \mathcal{S}_{\text{src}}} \|\mathbf{p}_\ell - \mathbf{f}(\mathbf{Y}_{\text{Rx},\ell}; \mathbf{w}')\|_2^2. \quad (26)$$

The solution to problem (SP2-1), \mathbf{w}'^* , can be obtained efficiently by using Adam algorithm [46], and we select $\mathbf{w}^{(0)*} = \mathbf{w}'^*$ as the proper initial point for the adaptation under the FL framework.

2) *Design of Adaptation Function:* Next, we design the adaptation function to adapt the solution to $\mathcal{T}(\mathcal{D}_{\text{src}})$ to $\mathcal{T}(\mathcal{D}_{\text{tar}})$, so that the objective function in (P1) can be optimized. Specifically, the adaptation function needs to satisfy two important conditions: *First*, the adaptation should not overfit the position estimator to the limited local datasets of the users; otherwise, the resulting position estimator may yield low positioning errors only around the anchors. *Second*, the adaptation should not be biased towards the local dataset(s) of one or few users; otherwise,

the resulting position estimator may only get low positioning errors for part of the users.

To satisfy the first condition while fully utilizing the limited target domain data, we employ a *fine-tuning technique* to set different adaptation rates for different parts of parameter vector $\mathbf{w}^{(t)}$, where the MLP of the position estimator is viewed as being composed of two components: The output layer of the MLP constitutes a *regressor* deriving the user's position from the *features* extracted by the other layers; and the other layers jointly constitute a *feature extractor*. We denote the parameters in $\mathbf{w}^{(t)} \in \mathbb{R}^{N_{\text{para}}}$ corresponding to the feature extractor and the regressor by $\mathbf{w}_{\text{feat}}^{(t)} \in \mathbb{R}^{N_{\text{feat}}}$ and $\mathbf{w}_{\text{reg}}^{(t)} \in \mathbb{R}^{N_{\text{reg}}}$, respectively, i.e., $\mathbf{w}^{(t)} = (\mathbf{w}_{\text{feat}}^{(t)}, \mathbf{w}_{\text{reg}}^{(t)})$ and $N_{\text{para}} = N_{\text{feat}} + N_{\text{reg}}$. We note that the feature extractor optimized for $\mathcal{T}(\mathcal{D}_{\text{src}})$ is also effective for $\mathcal{T}(\mathcal{D}_{\text{tar}})$ as the channels in both domains follow the same structure, i.e., (1)-(9), and the same DA beamforming configurations are employed. Therefore, the method for feature extraction needs little adaptation, and $\mathbf{w}_{\text{feat}}^{(t)}$ can be frozen or adapted with a very low rate η_{feat} . In contrast, the regressor has to be adapted substantially to handle the differences in the extracted feature values caused by the different environmental characteristics. Thus, we adapt the regressor with a large learning rate denoted by η_{reg} . Though the amount of local user data collected in \mathcal{D}_{tar} is small, the adaptation of the regressor can still be done effectively since the regressor only contains the output layer of the MLP with a small number of trainable parameters.

Besides, to satisfy the second condition, the aggregation function is expected to update the parameter vector along an unbiased gradient direction for minimization of the loss functions of all users, i.e., it should solve the following *target domain adaptation optimization* problem:

$$(\text{SP2-2}) : \min_{\{\mathcal{A}^{(t)}\}_t} \hat{\mathcal{L}}(\mathbf{w}^{(T)}) = \sum_{n=1}^U \hat{\mathcal{L}}_n(\mathbf{w}^{(T)}), \text{ s.t. (14e)-(14f), } \mathbf{w}^{(0)} = \mathbf{w}_{\text{src}}^*, \forall t = 1, \dots, T, \quad (27)$$

where $\hat{\mathcal{L}}_n(\mathbf{w}^{(T)})$ is defined in (11) and $\hat{\mathcal{L}}(\mathbf{w}^{(T)})$ denotes the total loss of all users w.r.t. $\mathbf{w}^{(T)}$. Therefore, in each positioning process t , the selected adaptation function $\mathcal{A}^{(t)*}$ should update $\mathbf{w}^{(t)}$ in the opposite direction of an unbiased estimate of $\nabla_{\mathbf{w}} \hat{\mathcal{L}}(\mathbf{w}^{(t)})$, which is denoted by $\mathbf{g}^{(t)}$.

To obtain this unbiased estimate, based on [47, Lemma 1], we can multiply the uploaded local update from user x , i.e., $\Delta \mathbf{w}_x^{(t)}$, with a *weight* which is in proportion to the size of user x 's local dataset, i.e., Q_x , and in inverse proportion to its scheduling probability, i.e., $\xi_x^{(t)}$, so that

$$\mathbb{E}_{x \sim \mathcal{M}(\xi^{(t)})} \left(\frac{Q_x}{Q \xi_x^{(t)}} \Delta \mathbf{w}_x^{(t)} \right) = -\nabla_{\mathbf{w}} \hat{\mathcal{L}}(\mathbf{w}^{(t)}) = -\mathbf{g}^{(t)}, \quad (28)$$

where $Q = \sum_{n=1}^U Q_n$ is the total size of all users' local datasets.

In summary, based on the fine-tuning technique and (28), the set of adaptation functions to

solve (SP2-2) can be designed as

$$\mathcal{A}^{(t)*}(\mathbf{w}^{(t)}, \Delta \mathbf{w}_x^{(t)}) = \mathbf{w}^{(t)} + \boldsymbol{\eta} \odot \left(\frac{Q_x}{Q_{\xi_x^{(t)}}} \Delta \mathbf{w}_x^{(t)} \right), \quad \forall t = 1, \dots, T, \quad (29)$$

where $\boldsymbol{\eta} = (\eta_{\text{feat}} \mathbf{1}_{N_{\text{feat}}}, \eta_{\text{reg}} \mathbf{1}_{N_{\text{reg}}})$ is the adaptation rate vector.

C. User Scheduling Probability Optimization

In the following, we handle the third challenge of (P1) by optimizing the user scheduling probability in each positioning process. Specifically, when optimizing $\boldsymbol{\xi}^{(t)}$, we consider two important factors as follows. *First*, in order to achieve fast convergence in FL, we consider the impact of $\boldsymbol{\xi}^{(t)}$ on the expected convergence rate. *Second*, to efficiently utilize the spectrum resource, we also evaluate the effect of $\boldsymbol{\xi}^{(t)}$ on the efficiency of gradient uploading.

Here, the convergence rate can be analyzed by extending [47, Lemma 2] to the FL framework under the proposed federated positioning protocol, as shown in the following proposition.

Proposition 3. *Given $\mathcal{A}^{(t)*}(\cdot)$ in (29), denote the optimal parameter vector for (SP2-2) by \mathbf{w}^* . The expected convergence rate for the t -th positioning process is characterized by*

$$\begin{aligned} \mathbb{E}(\hat{\mathcal{L}}(\mathbf{w}^{(t+1)}) - \hat{\mathcal{L}}(\mathbf{w}^*)) &\leq \mathbb{E}(\hat{\mathcal{L}}(\mathbf{w}^{(t)}) - \hat{\mathcal{L}}(\mathbf{w}^*)) - \boldsymbol{\eta}^\top \left(\mathbf{1} - \frac{L}{2} \boldsymbol{\eta} \right) \odot \mathbf{g}^{(t)\circ 2} \\ &\quad + \frac{L}{2} \sum_{n=1}^U \frac{1}{\xi_n^{(t)}} \cdot \left(\frac{Q_n}{Q} \right)^2 \cdot \left(\boldsymbol{\eta}^{\circ 2\top} \mathbb{E}(\mathbf{g}_n^{(t)\circ 2}) + \sigma_{\text{dp},n}^2 \|\boldsymbol{\eta}\|^2 \right) - \frac{L}{2} \boldsymbol{\eta}^{\circ 2\top} \mathbf{g}^{(t)\circ 2}. \end{aligned} \quad (30)$$

Here, $\mathbf{g}^{(t)}$ and $\mathbf{g}_n^{(t)}$ are the gradients of $\hat{\mathcal{L}}(\mathbf{w}^{(t)})$ and $\hat{\mathcal{L}}_n(\mathbf{w}^{(t)})$, respectively, as defined in (28) and below (11), L is the Lipschitz constant of the gradient, and $\boldsymbol{\eta}$ is defined in (29).

Proof: Please refer to Appendix B. ■

In (30), the only term related to $\boldsymbol{\xi}^{(t)}$ is the third term on the right-hand side of the inequality, which reflects the influence of $\boldsymbol{\xi}^{(t)}$ on the convergence rate and needs to be minimized. Proposition 3 reveals that the convergence rate is dependent on the powers of the gradients of the users and, to improve the rate of convergence, users having higher gradient powers should be scheduled with higher probabilities. However, as $\mathbb{E}(\mathbf{g}_n^{(t)\circ 2})$ is difficult to estimate by the BS or the users, we approximate it by $\mathbf{g}_n^{(t)\circ 2}$ as in [47]. Based on (30), for the scheduling probability of user n uploading in positioning process t , i.e., $\xi_n^{(t)}$, we define its *influence on convergence* as

$$Z_{\text{IC},n}^{(t)} = \left(\frac{Q_n}{Q} \right)^2 \cdot \left(\boldsymbol{\eta}^{\circ 2\top} \mathbb{E}(\mathbf{g}_n^{(t)\circ 2}) + \sigma_{\text{dp},n}^2 \|\boldsymbol{\eta}\|^2 \right). \quad (31)$$

Besides, we evaluate the influence of $\xi_n^{(t)}$ on the uploading efficiency by the ratio between *uploading duration* and *weighted uploading capacity*. The uploading duration is the time duration

for the uplink transmission of user n 's local update, and the uploading capacity is the maximum information on the optimization step of $\mathbf{w}^{(t)}$ contained in the uploaded gradient. We measure the uploading capacity based on the Shannon channel capacity formula [34]. Intuitively, the square of the uploaded gradient can be considered as the ‘‘transmit power’’, as a gradient with a larger squared norm can contain more information regarding the optimization step. The variance of the noise added to the gradient for DP training, i.e., $\sigma_{\text{dp},n}^2$, can be considered as the noise power of the ‘‘channel’’. Therefore, adding the weights for the learning rate and the data size, we define the *weighted uploading capacity* of user n 's uploading in positioning process t as

$$\Xi_n^{(t)} = \frac{Q_n}{Q} \cdot \log \left(1 + \frac{\boldsymbol{\eta}^{\circ 2\top} \mathbb{E}(\mathbf{g}_n^{(t)\circ 2})}{\sigma_{\text{dp},n}^2 \|\boldsymbol{\eta}\|_2^2} \right). \quad (32)$$

Then, the *influence on uploading efficiency* of $\xi_n^{(t)}$ is calculated as

$$Z_{\text{IE},n}^{(t)} = \frac{B_{\text{para}}}{R_n^{(t)} \Xi_n^{(t)}}, \quad (33)$$

where B_{para} denotes the total size of the gradient vector in bits, and $R_n^{(t)}$ denotes the uplink transmission rate from user n to the BS⁸. Here, exploiting channel reciprocity, $R_n^{(t)}$ can be obtained by user n based on the downlink frames from the BS for distributing $\mathbf{w}^{(t)}$.

To allow the BS getting enough information to optimize $\boldsymbol{\xi}^{(t)}$ while minimizing the overheads, during the federated adaptation phase, let each user n calculate $Z_{\text{IC},n}^{(t)}$ and $Z_{\text{IE},n}^{(t)}$ based on its local gradient and send these values to the BS. Since $Z_{\text{IC},n}^{(t)}$ and $Z_{\text{IE},n}^{(t)}$ are scalars and contain only the norms of the local gradients, the cost of uplink transmission and privacy leakage is negligible.

Jointly considering the convergence rate and uploading efficiency, we formulate a *joint convergence and efficiency scheduling optimization problem* as follows:

$$(\text{SP3}) : \min_{\boldsymbol{\xi}^{(t)}} \sum_{n=1}^U \left(\frac{1}{\xi_n^{(t)}} \cdot \frac{1}{\hat{Z}_{\text{IC}}^{(t)}} \cdot Z_{\text{IC},n}^{(t)} + \xi_n^{(t)} \cdot \frac{1}{\hat{Z}_{\text{IE}}^{(t)}} \cdot Z_{\text{IE},n}^{(t)} \right), \text{ s.t. (14g)}. \quad (34)$$

In (SP3), we employ two weight coefficients, i.e., $\hat{Z}_{\text{IC}}^{(t)}$ and $\hat{Z}_{\text{IE}}^{(t)}$, to rescale the impact of the convergence rate and the uploading efficiency. By this means, we prevent the optimization objective of (SP3) from being overly biased to either the convergence rate or the uploading efficiency given their different value ranges. Specifically, $\hat{Z}_{\text{IC}}^{(t)}$ and $\hat{Z}_{\text{IE}}^{(t)}$ are set as follows:

$$\hat{Z}_{\text{IC}}^{(t)} = \sum_{n=1}^U \frac{Z_{\text{IC},n}^{(t)}}{(\xi_n^{(t-1)})^2}, \quad \hat{Z}_{\text{IE}}^{(t)} = \sum_{n=1}^U Z_{\text{IE},n}^{(t)}, \quad \forall t = \{1, \dots, T\}, \quad (35)$$

⁸As user n can calculate $Z_{\text{IE},n}^{(t)}$ in (33) with $R_n^{(t)}$ being the uplink rate of the user, the proposed user scheduling algorithm can be extended to arbitrary band selection schemes for uplink transmission of local gradients.

Algorithm 2 Positioning Error Minimization Algorithm for HoloFed

- 1: Obtain $\{\mathbf{S}_{i,j}^*\}_{i,j}$ and $\{\mathbf{C}_i^*\}_i$ with Algorithm 1.
 - 2: Given $\{\mathbf{S}_{i,j}^*\}_{i,j}$ and $\{\mathbf{C}_i^*\}_i$, generate the training data in the source domain, i.e., \mathcal{D}_{src} .
 - 3: Solve problem (SP2-1) with the Adam algorithm [46] and obtain \mathbf{w}'^* as $\mathbf{w}^{(0)}$.
 - 4: **for** $t = 0, \dots, T$ **do**
 - 5: The BS distributes $\mathbf{w}^{(t)}$ to all the users, and each user n determines its uplink rate $R_n^{(t)}$ to the BS.
 - 6: Each user n calculates its local gradient by $\mathbf{g}_n^{(t)} = \nabla_{\mathbf{w}} \hat{\mathcal{L}}_n(\mathbf{w}^{(t)})$.
 - 7: Each user n calculates $Z_{\text{IC},n}^{(t)}$ and $Z_{\text{IE},n}^{(t)}$ based on (31) and (33) and sends them to the BS.
 - 8: The BS obtains the optimized scheduling probabilities, i.e., $\boldsymbol{\xi}^{(t)*}$, by solving (SP3).
 - 9: The BS randomly selects user x to upload its local update according to distribution $\mathcal{M}(\boldsymbol{\xi}^{(t)*})$.
 - 10: User x sends local update $\Delta \mathbf{w}_n^{(t)} = -\mathbf{g}_n^{(t)} + \boldsymbol{\varsigma}_n^{(t)}$ to the BS with $\boldsymbol{\varsigma}_n^{(t)}$ being the noise for differential privacy.
 - 11: The BS updates the global parameter vector as $\mathbf{w}^{(t+1)} = \mathcal{A}^{(t)*}(\mathbf{w}^{(t)}, \Delta \mathbf{w}_x^{(t)})$ based on (29).
-

with $\xi_n^{(0)} = 1/U$. It is straightforward to show that problem (34) is convex and has a unique optimal solution, which can be efficiently found by using convex optimization algorithms.

Finally, integrating the DA beamforming optimization, the initial point selection and adaptation function design, and the user scheduling probability optimization, the complete positioning error minimization algorithm for HoloFed is summarized as Algorithm 2.

V. SIMULATION RESULTS

In this section, the simulation is described and then our key simulation results are provided.

A. Simulation Setup

We establish a 3D coordinate system with its origin at the center of the RHS, its x-axis along the perpendicular direction of the RHS, and its z-axis pointing vertically upward. ROI \mathcal{P} is a cuboid region with its center at (10, 0, 0) m and 3D dimensions $(l_x, l_y, l_z) = (10, 10, 2)$ m. The distribution of the user positions, Γ^{U} , is a 3D uniform distribution within \mathcal{P} .

The RHS board is made of FR-4, which is a typical dielectric material used for printed circuit boards and has refractive index $n_r = 2.1$. The N_{B} bands of the system are centered at $f_i = (2 + 0.5i)$ GHz ($i \in \{1, \dots, N_{\text{B}}\}$) and the average wavelength of the center frequencies is denoted by λ_{avr} . The spacing between adjacent meta-elements is set to be $0.3\lambda_{\text{avr}}$.

As for position estimator $\mathbf{f}(\cdot; \mathbf{w})$, a 5-layered MLP with (1024, 512, 128) nodes in the three hidden layers is employed. Specifically, the output layer is treated as the regressor and the other layers are treated as the feature extractor, whose learning rates are set to $\eta_{\text{reg}} = 10^{-3}$

TABLE I
SIMULATION PARAMETERS

Parameter	Value	Parameter	Value	Parameter	Value	Parameter	Value
N_B	2	N_E	10×10	K	3	$\sigma_{\text{rms},i}$	$0.5 \mu\text{s}$
N_{SB}	8	v_{max}	20 km/h	Δ_t	$4 \mu\text{s}$	P_N	-174 dBm/Hz
F	4	P_{max}	1 mW	W	125 kHz	N_{sam}	10^4
T	400	U	2	N_{src}	10^5	$ \mathcal{B}_S , \mathcal{B}_C $	40
N_{out}	500	N_{in}	5	γ	0.95	β	0.1
δ_{dp}	10^{-5}	B_{para}	2.7×10^7 bits				

and $\eta_{\text{feat}} = 10^{-6}$, respectively. We consider the case with $U = 2$ users, where each user has $Q_n = 200$ labeled data in its local dataset, as default. Each data-label pair comprises a received signal matrix and the corresponding position label. A user obtains a position label when it is within 0.25 m of one of 10 anchors. For simulation, the positions of the 10 anchors are drawn from a uniform distribution within the ROI. The other default parameters are listed in Table I.

Besides, for the channel model in the source domain, the gains of each meta-element and user antenna are normalized as $g_{i,j}^E(\cdot) = g_{i,j}^U(\cdot) = 1$ ($\forall i, j$). Based on [35], we set $P_{\text{pap},i}(\boldsymbol{\theta})$ in (6) as a Laplacian function with zero mean and angular spread 10° in both azimuth and elevation, scaled by the average LoS power within the ROI. As for the exact channel model in the target domain, we assume a different gain pattern for the meta-elements, i.e., $g_{i,j}^E(\boldsymbol{\theta}) = \cos^{0.1}(\boldsymbol{\theta})$ and different multipath characteristics, i.e., $P_{\text{pap},i}(\boldsymbol{\theta})$ with mean angle 10° and angular spread 15° .

B. Results and Analyses

Firstly, we validate the CRLB gradient formulas in Proposition 2. Fig. 5 (a) shows the computational time of the proposed formulas and the finite difference (FD) method [48] and the maximum relative difference between them, which is calculated by $\max_{\ell} \{ |[\tilde{\boldsymbol{x}}]_{\ell} - [\boldsymbol{x}]_{\ell}| / \max(|[\boldsymbol{x}]_{\ell}|, 1) \}$. Here, $\tilde{\boldsymbol{x}}$ and \boldsymbol{x} denote the gradient vectors calculated by the proposed formulas and the FD method, respectively. The FD method is implemented based on the function `finitedifferences` provided by MATLAB[®]. As can be observed in Fig. 5, the proposed formulas can accurately determine the CRLB gradient with a significantly smaller computational time compared to the FD method. The maximum relative difference decreases with F since the CRLB decreases with F , and it increases with N_E since a larger number of variables incurs more numerical errors.

Secondly, we verify the efficiency of the proposed DA beamforming optimization algorithm in terms of CRLB minimization. To this end, we compare it with the benchmark ProxSARAH algorithm [43] through 30 independent trials. In each trial, $N_{\text{sam}} = 10^4$ random user positions

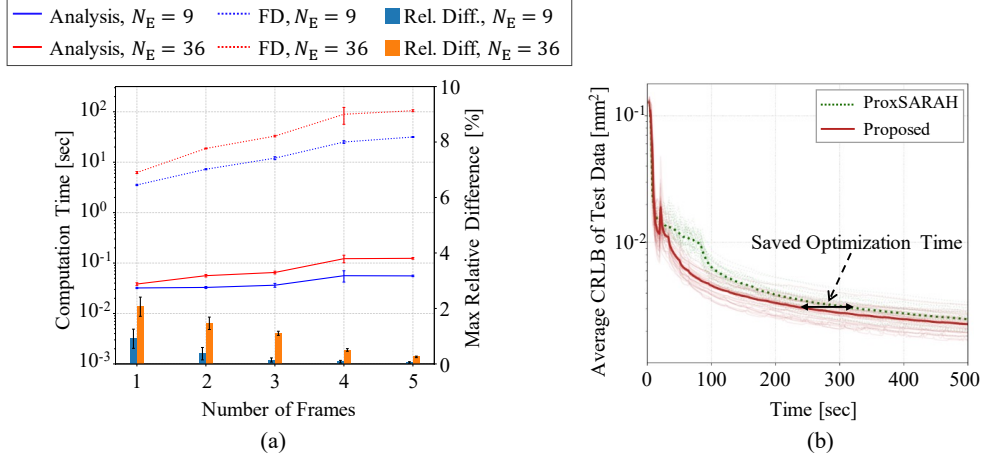


Fig. 5. (a) Computational time of the gradient formulas in Proposition 2 (*Analysis*) and the finite difference method provided by MATLAB[®] (*FD*) and their maximum relative difference (*Rel. Diff.*). The lines show the computational time, and the bars show the maximum element-wise relative difference. The error bars indicate the standard deviation over 30 independent trials. (b) Comparison between the proposed DA beamforming optimization algorithm and the benchmark ProxSARAH algorithm in [43]. The translucent and opaque lines show results for 30 individual trials and the average results, respectively.

within the ROI are sampled, where 90% of them are used for optimization and 10% of them are used as test data to evaluate the average CRLB. In Fig. 5 (b), it can be observed that, for the proposed algorithm, the average CRLB decreases with a faster rate than for ProxSARAH. This indicates that the proposed algorithm is more efficient due to the alternation between the DA variables. On average, the proposed algorithm saves 37.5% optimization time.

Thirdly, we verify the effectiveness of the proposed DA beamforming optimization in terms of the resulting MSE of positioning in the source domain. We compare the DA beamforming configuration optimized by Algorithm 1 with two baseline beamforming configurations from [14]:

- **Directional Beams (*DireBeam*):** The DA beamforming configuration generates focused beams scanning the ROI during the frames in the MMT phase in Sec. II-C1.
- **Random Beams (*RandBeam*):** The elements of $\{C_i\}_i$ are randomly distributed within $[0, 1]$, and the elements of $\{S_{i,j}\}_{i,j}$ take uniform values while satisfying the power constraint in (14c).

Fig. 6 shows the violin plot comparing the performance of different DA beamforming configurations in terms of the resulting MSE of positioning. The MSE of positioning is evaluated by the position estimator optimized for the source domain by solving (SP2-1) for each DA beamforming configuration. To reduce randomness, for each configuration, we evaluate the position estimator for 30 randomly generated test sets, and the resulting average MSE of positioning in each trial is shown by a dot in Fig. 6. As can be observed from Fig. 6, by using the DA beamforming configuration obtained with Algorithm 1, HoloFed can reduce the MSE of positioning by 57%

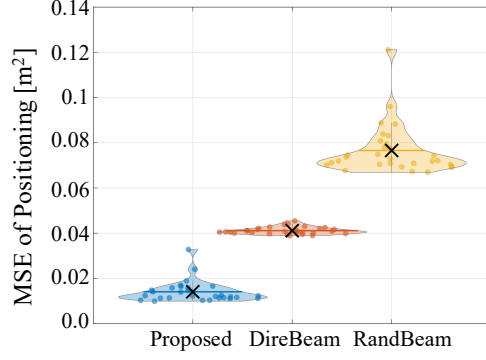


Fig. 6. Comparison of the MSE of positioning for different DA beamforming configurations.

and 78% on average compared to the DireBeam and RandBeam baselines, respectively.

Fourthly, we verify the efficiency of the proposed algorithm for user scheduling probability optimization in Sec. IV-C. We compare the results of the proposed algorithm with a state-of-the-art benchmark proposed in [47], which is referred to as channel-aware probabilistic scheduling (CAPS) algorithm. To facilitate a meaningful comparison, we compare the two scheduling algorithms for 6 typical situations, which are described in the caption of Figs. 7(a)-

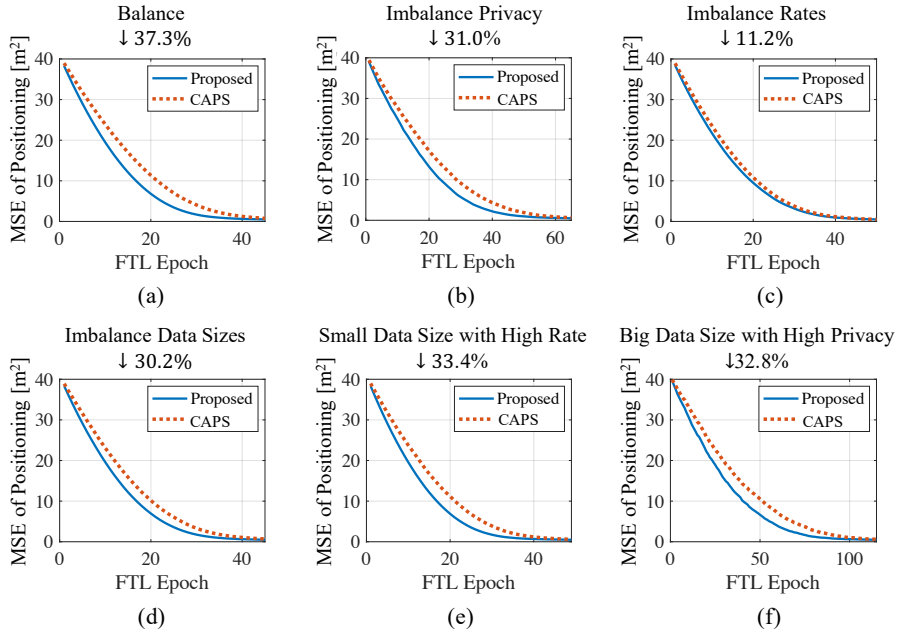


Fig. 7. Comparison of the adaptation performance in FL for the proposed user scheduling and the benchmark CAPS algorithms in 6 typical situations: (a) Users have balanced privacy leakage bounds, uplink rates, and local data sizes; (b) Users have imbalanced privacy leakage bounds, i.e., (100, 20); (c) Users have imbalanced uplink rates, i.e., (1, 0.2) Mbps; (d) Users have imbalanced local data sizes, i.e., (100, 20); (e) The user with a smaller data size has a higher uplink rate, i.e., rates (0.2, 1) Mbps and data sizes (100, 20); (f) The user with a larger data size has a tighter privacy leakage bound, i.e., local data size (200, 100) and privacy leakage bounds (5, 100).

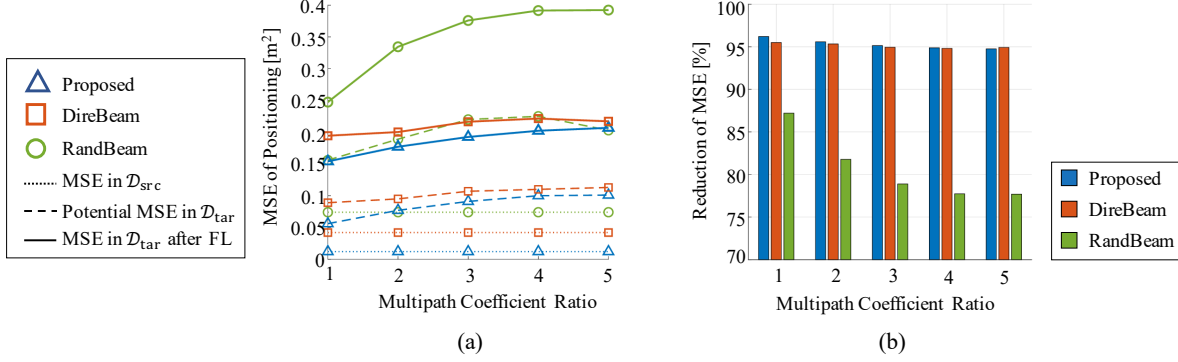


Fig. 8. (a) Comparison between the MSEs of positioning for different DA beamforming configurations. (b) Comparison between the reduction of the MSE due to the adaptation in FL for different DA beamforming configurations.

(f). Besides, to make the impact of scheduling on FL as prominent as possible, the parameters of the regressors in the MLPs are re-initialized randomly before the first positioning process.

Figs. 7(a)-(f) show the adaptation performance in terms of the MSE of positioning in different positioning processes, averaged over 30 independent trials. The proposed algorithm outperforms the benchmark algorithm in all considered situations. Moreover, we calculate the average relative gain in terms of the required number of epochs for the MSE of positioning to drop from its initial value to below 0.5 m^2 . The corresponding values are provided in the sub-headings of Figs. 7(a)-(f), e.g., $\downarrow 37.3\%$ in Fig. 7(a). It can be observed that the gains of the proposed algorithm are over 30% except for the imbalance rate situation. This verifies that the proposed algorithm is more efficient than the benchmark algorithm in terms of training the position estimator to adapt to the target domain.

Fifthly, we compare the MSE of positioning for the DA beamforming configurations obtained by Algorithm 1 and the DireBeam and RandBeam baselines for different levels of deviation between the source and target domains. We control the deviation by changing the multipath coefficient ratio from 1 to 5, which represents the ratio between the angular spread of $P_{\text{pap},i}(\theta)$ for the target domain and that for the source domain. The comparison focuses on three values: 1) the MSE in \mathcal{D}_{src} , 2) the potential MSE in \mathcal{D}_{tar} , and 3) the MSE in \mathcal{D}_{tar} after FL. Specifically, we evaluate the MSE in \mathcal{D}_{src} and the potential MSE in \mathcal{D}_{tar} by using supervised learning to train the position estimator with 10^5 labeled data in \mathcal{D}_{src} and \mathcal{D}_{tar} , respectively; and we evaluate the MSE in \mathcal{D}_{tar} after FL by using the position estimator adapted to \mathcal{D}_{tar} with Algorithm 2. Fig. 8 (a) verifies that the DA beamforming configuration obtained with the proposed algorithm leads to the lowest potential MSE in \mathcal{D}_{tar} and the lowest MSE in \mathcal{D}_{tar} after FL.

Finally, we verify the performance of HoloFed in adapting to diverse environments by showing that the MSE of positioning in different target domains can be effectively reduced through the adaptation in the proposed protocol. Fig. 8 (b) shows the reduction of the MSE of positioning *before* and *after* the adaptation. It can be observed that for different levels of deviation between the target and source domains, the federated adaptation consistently reduces the MSE of positioning by around 95% for both the proposed and the DireBeam DA beamforming configurations.

VI. CONCLUSION

In this paper, we have proposed HoloFed, a user positioning system based on MB-RHS and FL which can adapt to diverse practical environments. We have formulated a positioning error minimization problem for HoloFed and solved it by decomposing the problem into three subproblems. First, we derived the CRLB of the positioning error and utilized it for optimization of the DA beamforming configuration of the RHS. Second, we exploited transfer learning to select the initial point and adaptation function in FL. Third, we proposed a user scheduling probability optimization algorithm, jointly considering the convergence rate and uploading efficiency of FL.

Simulation results have shown that the proposed DA beamforming optimization algorithm can reduce the computation time required by ProxSARAH by 37.5% and result in a 57% lower MSE of positioning compared to DireBeam baseline. Moreover, the proposed user scheduling optimization algorithm achieves a 11% ~ 37% lower average MSE in FL process compared to the benchmark CAPS. Furthermore, we showed that HoloFed can adapt to diverse environments via federated adaptation, which can reduce the MSE of positioning by around 95%.

APPENDIX A

COMPONENTS OF THE GRADIENTS OF THE CRLB

With the help of [49, Eqs. (36)-(40)], the notations in (25) can be derived as

$$\begin{aligned} \mathbf{A}_{i,j,u,v}^s &= ([\bar{\boldsymbol{\zeta}}_{i,u}]_{(j-1)F+1:jF} \otimes \mathbf{1}_K^\top) \odot \mathbf{R}_{F \times K} \left(\left(([\dot{\mathbf{H}}_{i,v}^{\text{LoS}}]_{(j-1)F+1:jF} \odot \mathbf{C}_i) [\odot] (\mathbf{B}_{i,j} \otimes \mathbf{1}_F) \right) \mathbf{1}_{N_E} \right), \\ \mathbf{B}_{i,j,u,v}^s &= -([\bar{\boldsymbol{\zeta}}_{i,u}]_{(j-1)F+1:jF} \otimes \mathbf{1}_K^\top) \odot \mathbf{R}_{F \times K} \left(\left([\mathbf{K}_{\text{ft},i}]_{(j-1)F+1:jF} [\odot] \left((\mathbf{C}_i [\odot] \mathbf{B}_{i,j} \otimes \mathbf{1}_F) (\mathbf{V}_i \mathbf{T}_i^{\text{H}}) \right) \right) \boldsymbol{\zeta}_{i,v} \right). \end{aligned}$$

Here, $\boldsymbol{\zeta}_{i,u} = \mathbf{A}_i^{-1} (\dot{\mathbf{H}}_{i,u}^{\text{LoS}} \odot \mathbf{T}_i) \mathbf{1}_{N_E} \in \mathbb{C}^{N_{\text{SB}} F \times 1}$, operator $[\odot]$ denotes the *penetrating face product*, and function $\mathbf{R}_{F \times K}(\cdot)$ reshapes the vector in the argument to an $F \times K$ matrix. Similarly, the corresponding notations for $\partial \mathbf{I}_{\text{FIM}}(\mathbf{p}) / \partial \mathbf{C}_i$ can be obtained as

$$\mathbf{A}_{i,u,v}^c = \sum_{j=1}^{N_{\text{SB}}} ([\bar{\boldsymbol{\zeta}}_{i,u}]_{(j-1)F+1:jF} \otimes \mathbf{1}_{N_E}^\top) \odot [\dot{\mathbf{H}}_{i,v}^{\text{LoS}}]_{(j-1)F+1:jF} \odot (\mathbf{S}_{i,j} \mathbf{B}_{i,j}),$$

$$\mathbf{B}_{i,u,v}^c = - \sum_j^{N_{SB}} ([\bar{\boldsymbol{\zeta}}_{i,u}]_{(j-1)F+1:jF} \otimes \mathbf{1}_{N_E}^\top \odot (\mathbf{S}_{i,j} \mathbf{B}_{i,j})) \odot \mathbf{R}_{F \times N_E} \left([\mathbf{K}_{ft,i}]_{(j-1)F+1:jF} [\odot] ((\mathbf{V}_i \mathbf{T}_i^H) \otimes \mathbf{1}_F) \boldsymbol{\zeta}_{i,v} \right).$$

APPENDIX B

PROOF OF PROPOSITION 3

Based on [47, Lemma 2], given parameter vectors denoted by \mathbf{a} and \mathbf{b} , it can be derived that

$$\hat{\mathcal{L}}(\mathbf{a}) \leq \hat{\mathcal{L}}(\mathbf{b}) + \nabla_{\mathbf{b}} \hat{\mathcal{L}}(\mathbf{b})^\top (\mathbf{a} - \mathbf{b}) + \frac{L}{2} \|\mathbf{a} - \mathbf{b}\|_2^2. \quad (36)$$

Substituting $\mathbf{a} = \mathbf{w}^{(t+1)}$, $\mathbf{b} = \mathbf{w}^{(t)}$, and $\nabla_{\mathbf{b}} \hat{\mathcal{L}}(\mathbf{b}) = \mathbf{g}^{(t)}$ into (36), it can be shown that

$$\hat{\mathcal{L}}(\mathbf{w}^{(t+1)}) \leq \hat{\mathcal{L}}(\mathbf{w}^{(t)}) + (\mathbf{g}^{(t)})^\top (-\boldsymbol{\eta} \odot \hat{\mathbf{g}}^{(t)}) + \frac{L}{2} \|\boldsymbol{\eta} \odot \hat{\mathbf{g}}^{(t)}\|^2. \quad (37)$$

where $\hat{\mathbf{g}}_x^{(t)} = -Q_x \Delta \mathbf{w}_x^{(t)} / (Q \xi_x^{(t)})$. Taking the expectation of both sides of (37), we obtain

$$\begin{aligned} \mathbb{E} \left(\hat{\mathcal{L}}(\mathbf{w}^{(t+1)}) \right) &\leq \mathbb{E} \left(\hat{\mathcal{L}}(\mathbf{w}^{(t)}) \right) - (\mathbf{g}^{(t)})^\top \boldsymbol{\eta} \odot \mathbb{E}(\hat{\mathbf{g}}_x^{(t)}) + \frac{L}{2} (\boldsymbol{\eta})^{\circ 2 \top} \mathbb{E}((\hat{\mathbf{g}}_x^{(t)})^{\circ 2}) \\ &= \mathbb{E} \left(\hat{\mathcal{L}}(\mathbf{w}^{(t)}) \right) - (\boldsymbol{\eta})^\top (\mathbf{g}^{(t)}) \odot \mathbb{E}(\hat{\mathbf{g}}_x^{(t)}) + \frac{L}{2} (\boldsymbol{\eta})^{\circ 2 \top} \left((\mathbb{E}(\hat{\mathbf{g}}_x^{(t)}))^{\circ 2} + \mathbb{V}(\hat{\mathbf{g}}_x^{(t)}) \right) \\ &\stackrel{(a)}{=} \mathbb{E} \left(\hat{\mathcal{L}}(\mathbf{w}^{(t)}) \right) - (\boldsymbol{\eta})^\top \left(\mathbf{1} - \frac{L}{2} \boldsymbol{\eta} \right) \odot (\mathbf{g}^{(t)})^{\circ 2} + \frac{L}{2} (\boldsymbol{\eta})^{\circ 2 \top} \mathbb{V}(\hat{\mathbf{g}}_x^{(t)}), \end{aligned} \quad (38)$$

where (a) is because $\hat{\mathbf{g}}^{(t)}$ is an unbiased estimate of $\mathbf{g}^{(t)}$, and $\mathbb{V}(\hat{\mathbf{g}}_x^{(t)})$ is the covariance of $\hat{\mathbf{g}}_x^{(t)}$:

$$\begin{aligned} \mathbb{V}(\hat{\mathbf{g}}_x^{(t)}) &= \mathbb{E}((\hat{\mathbf{g}}_x^{(t)} - \mathbf{g}^{(t)})^{\circ 2}) = \mathbb{E}(\hat{\mathbf{g}}_x^{(t)\circ 2}) - \mathbf{g}^{(t)\circ 2} = \sum_{n=1}^U \xi_n^{(t)} \left(\left(\frac{Q_n}{Q \xi_n^{(t)}} \right)^2 \mathbb{E}(\mathbf{g}_n^{(t)\circ 2}) + \left(\frac{Q_n}{Q \xi_n^{(t)}} \right)^2 \mathbb{E}(\boldsymbol{\varsigma}_n^{(t)\circ 2}) \right) \\ &= \sum_{n=1}^U \frac{1}{\xi_n^{(t)}} \cdot \left(\frac{Q_n}{Q} \right)^2 \cdot (\mathbb{E}(\mathbf{g}_n^{(t)\circ 2}) + \sigma_{\text{dp},n}^2 \mathbf{1}). \end{aligned} \quad (39)$$

Subtracting $\mathbb{E}(\hat{\mathcal{L}}(\mathbf{w}^*))$ from both sides of (38), then (30) in Proposition 3 is proven.

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Response to Reviews of the Article #1570878869: “HoloFed: Environment-Adaptive Positioning via Multi-band Reconfigurable Holographic Surfaces and Federated Learning”

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The authors would like to thank the editor and the reviewers for their constructive suggestions and positive comments, which have helped improve the content and presentation of the manuscript. We have revised the manuscript based on the reviewers' comments and suggestions. In particular, we clearly explain the four distinct novelties of this paper compared to the related conference paper accepted by IEEE ICC 2023, where only a small part of this work has been reported. The detailed explanation can be found in our responses to Comments C and D of Review 2. We have added some necessary references to support our arguments, namely [23], [24], [37], [38], [45]. We selected these references based on their relevance and declare that none of the newly added references are our previous works.

In the following, we provide point-by-point responses to the raised comments of all the reviewers. The detailed revisions are highlighted in the manuscript and summarized below.

- **Section I:** 1) In the fifth paragraph, we have added two references, i.e., [23], [24], for existing designs and prototypes of multi-band (MB) reconfigurable holographic surfaces (RHSs) and reconfigurable intelligent surfaces (RIS) to support the feasibility of MB-RHS. 2) We have added a statement following our main contributions listed in the introduction to explain the novelty of this JSAC manuscript compared to its conference version presented in IEEE ICC 2023.
- **Section II:** 1) In Sec. II-A2, we have revised the description of the user position distribution. 2) Besides, below (6), we have added an explanation that our channel model can account for the passive interference among users. 3) We have revised the first paragraph of Sec. II-C1 to emphasize the reason why the transmissions in multiple bands are performed sequentially. We added a footnote, namely Footnote 5, to clarify that the proposed algorithm can also be adapted to the parallel transmission case. 4) In the second paragraph of Sec. II-C1, we have added an analysis on the time complexity of the multi-pattern transmission (MMT) phase. 5) In the second paragraph of Sec. II-C3, we have added an footnote, namely Footnote 6, to clarify that

short-ranging techniques for user positioning near the anchors can be readily supported by the existing IoT infrastructures. We have also added some references, i.e., [37], [38], to support this claim. 6) Below (11), we have added an explanation for how the proposed system utilizes the position labels provided by the anchors and what the benefit is. 7) In the fourth paragraph of Sec. II-C3, we have added a footnote, namely Footnote 7, to explain how the users can select the band to transmit their local gradient and that the proposed algorithm supports this. 8) At the end of Sec. II-C, we have added a remark, namely Remark 2, to clarify that there are no active user interference in HoloFed.

- **Section IV:** 1) At the end of Sec. IV-A, we have added two remarks, namely Remarks 3 and 4. In Remark 3, we explain how user interference impacts the CRLB for positioning. In Remark 4, we explain how using multiple bands helps lower the CRLB. 2) In the second paragraph of Sec. IV-A2, we have revised the description of the proposed algorithm to clarify why it contains an outer loop and an inner loop. 3) Besides, in the third paragraph of Sec. IV-B, we have added an explanation for why the source and target domains are different from each other, and why transfer learning can help the adaptation of the position estimator to the target domain. We have added a new supportive reference, namely [45], for transfer learning. 4) In the second paragraph of Sec. IV-B2, we have further clarified the working principle of the fine-tuning technique adopted in the proposed algorithm. 5) Moreover, below Proposition 3 in Sec. IV-C, we have added an explanation for why we adopt a gradient power based criterion in the user scheduling algorithm. 6) Furthermore, below (33) in Sec. IV-C, we have added a footnote, namely Footnote 8, to clarify that the proposed algorithm allows the users to perform band selection, supporting our statement in Footnote 7.
- **Section V:** 1) We have enlarged the figures to enhance the legibility of their text elements. 2) In the third paragraph of Sec. V-A, we have revised the description of the position distribution of the anchors to make it clearer. 3) In the description of Fig. 6 in Sec. V-B, we explain now the meaning of the dots. 4) In the description of Fig. 7 in Sec. V-B, we have revised the text to clarify the target criterion we considered in the comparison.

RESPONSE TO REVIEW 1

A. The work is very interesting and presents an interesting technical endeavor using many different techniques to solve the issues arising from their proposal. The necessary optimization problems are solved using appropriate techniques.

Response: We thank the reviewer for the positive feedback on our manuscript. We have tried our best to address all comments of the reviewer in order to further enhance the quality and clarity of our paper.

B. My main upset regards the data used in the learning process. The authors claim to present a proposal independent of using any extra positioning infrastructure and relying solely on the RHS, a data-exchanging protocol, and an ML-based regressor. However, all the presented algorithms and results depend on an extra infrastructure for the positioning labels used in the learning process. If some beacons are used to estimate the data labels when users are sufficiently close, a parallel/concurrent positioning capacity is necessary? This eliminates many of the claimed benefits of the proposal. Moreover, can you affirm that this allows us to effectively train the regression for the signals emitted by the RHS? The last is characterized by band-dependent directional patterns, the physical principle supporting the proposed positioning protocol.

Response: We thank the reviewer for this comment. In the comment, two questions are raised by the reviewer: 1) *Does using beacons to estimate the data labels of the users rely on an extra infrastructure with parallel positioning capacity and thus eliminate the claimed benefits of the proposal?* 2) *Why does using the data labels near the beacons allow us to effectively train the regression for the signals emitted by the RHS?* In the following, we address these two questions sequentially.

1) Justification for using “beacons” to estimate the data labels: In this paper, as we have described in the second paragraph of the introduction, our motivation is to provide high precision positioning service in a large region of interest (ROI), without deploying extra infrastructure, e.g., a massive number of RFID tags to cover the ROI. Nevertheless, this does not mean that we do not use existing infrastructure. As we have explained in our main contributions in the introduction, we have designed HoloFed to deliver low positioning error and high environmental adaptivity without compromising the users’ privacy. To achieve this, we assume that the users possess a small number of position labels in deployment environment of HoloFed, so that the

users can adapt the position estimator of HoloFed to the environment. To obtain such position labels, the users are required to employ short-range positioning techniques that operate in parallel with HoloFed. This requirement does not incur extra infrastructure requirements or eliminate the benefits and practicability of our proposal, and hence is compatible with our motivation. This can be further justified as follows.

In this paper, as the reviewer has pointed out, we assume that the users can collect their position labels as the data labels when they are near a small number of “beacons” in the ROI. We refer to these “beacons” as *anchors*, as described in Sec. II-C3. To obtain its position when near an anchor, a user is required to be able to perform short-range positioning in parallel with HoloFed. This requirement can be satisfied by already existing infrastructure; therefore, it does not require additional infrastructure. Specifically, when a user conducts near-field communication with the surrounding infrastructure (e.g., smart gate machines, point-of-sale machines, and RFID tags), the infrastructure can be considered as anchor, and a position label can be obtained since the position of the infrastructure is known and the user is in its proximity [37]. Infrastructures that support such short-range positioning have already been widely deployed in practice because of the prevalence of the Internet of Things (IoT) [38].

However, in general, existing infrastructure can only provide a small number of anchors that are not densely deployed, and thus such short-range positioning capability cannot provide consistent high-precision positioning services to users within a large ROI. In comparison, our proposed HoloFed can provide high-precision positioning services within a large ROI, and thus the assumption that users can obtain position labels in the proximity of a small number of anchors does not degrade the benefits of our proposed approach. Actually, one of the merits of our proposed HoloFed is that HoloFed can effectively merge the information provided by RHS-based signal transmission with the limited labels provided by existing short-range positioning techniques to achieve high-precision large-range positioning with environmental adaptability.

To clarify this in the manuscript, we have added a footnote in the second paragraph of Sec. II-C3, where we justify that short-range positioning techniques are supported by the existing and widely deployed IoT infrastructures. The added footnote is reproduced below.

“⁶Here, an anchor refers to a location where the users can obtain their position labels based on short-range positioning techniques. Such short-range positioning techniques can be readily supported by near-field communication (NFC) of the users with the existing Internet of Things (IoT) infrastructure [37], [38].”

Besides, we have added an explanation in Sec. II-C3 to clarify that with the help of the

position labels collected near a small number of anchors, HoloFed can achieve high-precision large-range positioning with environmental adaptability. The added part is reproduced below.

“Exploiting the local gradient of the positioning loss w.r.t. the position labels collected near a small number of anchors, HoloFed adapts its position estimator to the actual deployment environment for achieving large-range positioning with high precision.”

2) *Explanation for why these data labels can be used to train the regression effectively:*

Though users can only collect position labels near a limited number of anchors, HoloFed can use them effectively to train the regression of user positions, based on the fine-tuning technique utilized in the proposed algorithm. To achieve this goal, as described in Sec. IV-B, we first consider a *source domain* where the environmental characteristics of the channel model are known, in contrast to the *target domain* where HoloFed is deployed and the environmental characteristics are unknown. As described in Sec. IV-B, based on transfer learning, our procedure to train the regression can then be decomposed into two steps: In Step 1, we train the regression in the source domain; In Step 2, we adapt the regression to the target domain. For the first step, as the channel model in the source domain is known, sufficient training data across the entire ROI is obtained, which can be utilized to train the regression for the band-dependent directional patterns of the emitted signals. Specifically, we adopt a multilayer perceptron (MLP) with trainable parameters for position estimation to perform this regression.

Then, as for the second step, based on the fine-tuning technique, we decompose the position estimator into two parts: a feature extractor and a regressor. The regressor is composed of the output layer of the MLP, which contains a few trainable parameters; while the other layers constitute the feature extractor, which contains most of the trainable parameters of the MLP. Due to the intrinsic similarity between the source and target domains, which originates from the same signal propagation models and digital-analog beamforming configurations, the method to extract features from the signals should be similar in the source and target domains; while the values of the extracted features can be distinct due to different environmental characteristics. Therefore, the parameters of the feature extractor can be frozen or adapted with a very small learning rate, but the parameters of the regressor need to be re-trained substantially so that they can adapt to the extracted feature with different values. Nevertheless, since the regressor only contains a few trainable parameters, the adaptation can be effectively performed with only a small number of data-label pairs in the target domain.

This fine-tuning technique is also widely adopted in transfer learning, and enables effective

training of classification and regression functions with insufficient data-label pairs [45]. Consequently, by using the algorithm proposed in Sec. IV-B, we ensure that using the collected position labels near the anchors allows us to train the regression for the emitted signals effectively. The effectiveness of the regression is validated by Figs. 7 and 8, where it can be observed that after training with the collected position labels, the position regression adapts to the target domain.

To address this point in the manuscript, we have made three key revisions to explain why the training is effective. Firstly, we have revised the third paragraph of Sec. IV-B, where we emphasize that the similarity between the source and target domains lays the foundation for transfer learning. The revised part is reproduced below.

“Nevertheless, due to the intrinsic similarities between \mathcal{D}_{tar} and \mathcal{D}_{src} (e.g., the underlying signal propagation models and DA beamforming configurations are identical), transfer learning can be used efficiently to adapt \mathbf{w}^* to \mathcal{D}_{tar} [45].”

Secondly, we have revised the second paragraph of Sec. IV-B2. We explain now that the feature extractor does not need re-training because the feature extraction method remains effective in the target domain:

“We note that the feature extractor optimized for $\mathcal{T}(\mathcal{D}_{\text{src}})$ is also effective for $\mathcal{T}(\mathcal{D}_{\text{tar}})$ as the channels in both domains follow the same structure, i.e., (1)-(9), and the same DA beamforming configurations are employed. Therefore, the method for feature extraction needs little adaptation, and $\mathbf{w}_{\text{feat}}^{(t)}$ can be frozen or adapted with a very low rate η_{feat} .”

Finally, in the second paragraph of Sec. IV-B2, we have clarified that the regressor can be adapted effectively with limited training data-label pairs because it comprises only a small number of trainable parameters. The new text is reproduced below.

“In contrast, the regressor has to be adapted substantially to handle the differences in the extracted feature values caused by the different environmental characteristics. Thus, we adapt the regressor with a large learning rate denoted by η_{reg} . Though the amount of local user data collected in \mathcal{D}_{tar} is small, the adaptation of the regressor can still be done effectively since the regressor only contains the output layer of the MLP with a small number of trainable parameters.”

C. Besides, saying that the user is close to an anchor is too vague; although the authors set this distance to .25 m in the simulation, they spread anchors over the whole cuboid ROI. Saying that ten anchors are uniformly distributed in the ROI is very imprecise. Do you say they are drawn using a uniform distribution or have a specific 3D arrangement?

Response: We thank the reviewer for this comment and the careful reading. The simulation setup mentioned by the reviewer is in the third paragraph of Sec. V-A. In the simulation, the positions of the anchors are drawn from a uniform distribution within the ROI. A user is considered to be close to an anchor when it is within 0.25 m of the anchor’s location. If a user is close to an anchor, it can obtain its current position, which can be used as label for the federated adaptation of the position estimator.

To clarify this in the manuscript, we have revised the description in Sec. V-A corresponding to this comment. The text is reproduced below.

“Each data-label pair comprises a received signal matrix and the corresponding position label. A user obtains a position label when it is within 0.25 m of one of 10 anchors. For simulation, the positions of the 10 anchors are drawn from a uniform distribution within the ROI.”

D. This links to another aspect that needs to be better explained. For the presented solution to use transfer learning, the source and target ROI should be different, authors try to differentiate them using the multipath-coefficient ratio in PaP, but this reflects changing the RHS instead of the domain. However, results use the same domain. I need help finding where transfer learning is applied.

Response: We thank the reviewer for this comment. In this paper, the different environmental characteristics, including the multipath-coefficient ratios, i.e., $P_{\text{pap},i}(\boldsymbol{\theta})$, of the source and target domains reflect the differences in the channel environments of the source and target ROIs. This is described in the third paragraph of Sec. IV-B, where we provide the reason for using transfer learning. To be specific, this can be justified as follows.

In transfer learning, a *domain* represents the joint distribution of the data-label pairs, and the source and target domains that are different need to have different joint distributions of the data-label pairs [45]. In this paper, our source and target domains satisfy this requirement since the gain patterns of the meta-elements and the power-angle profiles in the source and target domains are different, as explained in the remark at the end of Sec. II-B and the simulation

setup in Sec. V-A. These differences result in channel models with different environmental characteristics, which makes the “data”, i.e., the received signals of a user, to have a different distribution with respect to the user position, causing a position estimator trained in the source domain to be not effective in the target domain. Nevertheless, due to the intrinsic similarities between the source and target domains (e.g., the same signal propagation models and digital and analog (DA) beamforming configurations), transfer learning can be applied to adapt the position estimator to the target domain efficiently. For this reason, we propose a transfer learning based algorithm to adapt the position estimator trained in the source domain to the target domain, minimizing the positioning error.

To clarify this point in the manuscript, we have added a corresponding explanation in Sec. IV-B, which is reproduced below.

“To obtain \mathbf{w}^{*l} , certain environmental characteristics need to be assumed for \mathcal{D}_{src} , which are generally different from those in \mathcal{D}_{tar} . This results in different joint distributions for the user positions and received signals in the two domains, and hence \mathbf{w}^{*l} is not valid in \mathcal{D}_{tar} . Nevertheless, due to the intrinsic similarities between \mathcal{D}_{tar} and \mathcal{D}_{src} (e.g., the underlying signal propagation models and DA beamforming configurations are identical), transfer learning can be used efficiently to adapt \mathbf{w}^{*l} to \mathcal{D}_{tar} [45].”

E. Figures should be enlarged to ease reading.

Response: We thank the reviewer for this comment. As suggested, we have enlarged all figures in our manuscript, enhancing the legibility of the text elements of these figures.

F. Some phrases need a better explanation, like “the positions of a user for different positioning processes are independent and identically distributed.”

Response: We thank the reviewer for the careful reading. In Sec. II-A2, the phrase pointed out by the reviewer indicates that in each positioning process, the positions of the users are assumed to be independent and identically distributed random variables. To clarify this in the manuscript, we have rewritten the relevant text in Sec. II-A2. The new text is reproduced below.

“Moreover, we assume that the positioning processes take place periodically. In each positioning process, the positions of the users are assumed to be independent and identically distributed random variables, each following a distribution Γ^U , i.e., $\mathbf{p} \sim \Gamma^U$.”

G. The sentence “we assume that in each positioning process, only one user is selected to upload its local update over a single band, so that the occupation of the time-spectrum resources for FL is minimized” makes one wonder if there would be beneficial to select the user band or using multiples bands for a user in the positioning task.

Response: We thank the reviewer for this comment. This comment involves two aspects.

Firstly, in the positioning task of the proposed HoloFed system, all the bands are utilized as described in Sec. II-C, which is beneficial for high positioning precision. Specifically, in the first phase of the protocol, the base station (BS) transmits in all N_B bands sequentially, generating multiple beam patterns in each band. Each user receives the signals in the N_B bands and jointly uses them to estimate its position. To clarify the benefit of using multiple bands for positioning, we have added a new remark below Proposition 1 in Sec. IV-A1, which is reproduced below.

“ **Remark 4:** According to Proposition 1, the benefits of using multiple bands for positioning are two-fold. *Firstly*, the Fisher information from multiple bands adds up, leading to a lower value for the CRLB. *Secondly*, the fact that signals received in different bands are generally less correlated (due to less correlated multipath gains) also contributes to a lower CRLB. This can be shown by deriving the Fisher information of two correlated signals based on [41, Eq. (3.31)].”

Secondly, with regard to the sentence that the reviewer referred to, we assume that a user uploads its local update using a single band, which means that in the federated adaptation phase, a user uses one band to transmit its local gradient to the BS for training the position estimator. This assumption lowers the hardware requirements for the BS and users as they do not require radio-frequency (RF) chains that support simultaneous signal transmission and reception capability in multiple bands. As the reviewer has pointed out, the band where a user conducts this transmission can be selected by the user. If the BS and the user have the capability of simultaneously transmitting and receiving in multiple bands, respectively, the user can even upload its local gradient over multiple selected bands. Both options are beneficial for the uplink transmission rate of the user, at the cost of incurring additional overhead for band selection and/or higher requirements on the RF chains. To keep the description of the user scheduling algorithm concise and focused, we do not explicitly include such band selection in our manuscript.

Nevertheless, it is worth mentioning that the user scheduling probability optimization proposed in our paper can easily be adapted to these scenarios by substituting the estimated uplink

transmission rate with that of the selected single or multiple bands. To clarify this in the manuscript, we have added a footnote in the sentence mentioned by the reviewer in Sec. II-C3. The footnote is reproduced below.

“⁷The band used for uploading can be selected by the user for rate maximization. Even multiple bands can be used if the user and the BS can support it. The proposed algorithm can be modified to accommodate such cases, as described in Sec. IV-C.”

Besides, to emphasize that our proposed scheme can be adapted to the cases of selecting single and multiple bands, we have added a footnote in the discussion of (33) in Sec. IV-C, which is reproduced below.

“⁸As user n can calculate $Z_{IE,n}^{(t)}$ in (33) with $R_n^{(t)}$ being the uplink rate of the user, the proposed user scheduling algorithm can be extended to arbitrary band selection schemes for uplink transmission of local gradients.”

H. The link between the gradient power and the necessary capacity, although intuitive, is not necessarily correct. As a suggestion, an information-based criterion could be more effective.

Response: We thank the reviewer for this comment. In our paper, we choose the criterion of gradient power not only because of its intuitive relation to the necessary capacity, but also based on the derivation in Proposition 3 in Sec. IV-C. In the proof of Proposition 3 in Appendix B, we have proven that the convergence rate depends on the gradient power of user n , i.e., $g_n^{(t)\circ 2}$. Moreover, from (30), we observe that in order to enhance the convergence rate, it is preferable to allocate users having higher gradient powers with larger scheduling probabilities, i.e., $\xi_n^{(t)}$. Based on this derivation and observation, we have designed the criterion for a user’s influence on convergence based on its gradient power. Furthermore, the effectiveness of using the gradient power based criterion is also validated by our simulation results in Fig. 7. To clarify the reason why we adopt a gradient power based criterion, we have emphasized this below Proposition 3 in Sec. IV-C. The corresponding text is reproduced below.

“Proposition 3 reveals that the convergence rate is dependent on the powers of the gradients of the users and, to improve the rate of convergence, users having higher gradient powers should be scheduled with higher probabilities.”

As the reviewer has kindly pointed out, an information-based criterion for the users’ impact on convergence could possibly be more effective than our method, provided that a tighter bound on the convergence rate can be derived using information-based analytical approaches. Nevertheless, since we can only devote a limited amount of space to achieving environmental adaptivity in

MB-RHS-based ISAC systems in a privacy-preserving manner, we would prefer to leave further exploration of a more efficient user scheduling algorithm for future work.

I. In the Caption of figure 5, please correct “differencemethod”.

Response: We thank the reviewer for the careful reading. We have corrected the typo pointed out by the reviewer in the caption of Fig. 5.

J. The graphs in Figure 6 do not seem to consider 20% of the 10^4 samples said to be used in the tests.

Response: We thank the reviewer for the careful reading and the comment. In Fig. 6, each of the 30 dots in the violin shape for each configuration represents the average MSE of positioning for an independent trial conducted on a randomly generated test set, rather than a single sample from the test set. Therefore, each dot in a violin shape represents 20% of the 10^4 samples in the test. To clarify this issue in the revised manuscript, we have added an explanation for this setup and the physical meaning of each dot in the discussion of Fig. 6 in Sec. V-B. The corresponding text is reproduced below.

“ To reduce randomness, for each configuration, we evaluate the position estimator for 30 randomly generated test sets, and the resulting average MSE of positioning in each trial is shown by a dot in Fig. 6.”

RESPONSE TO REVIEW 2

A. *The manuscript is well written and has a good readability. However, some minor issues need to be fixed, images are not easy to read and some sentences need to be rewrote.*

Response: We thank the reviewer for this comment. To address this comment, we have enlarged all figures in our manuscript, enhancing the legibility of the text elements of these figures. Besides, we have checked the manuscript and we have rewritten some sentences to improve their readability.

For example, in Sec. II-A2, we have rewritten the sentence that describes the distribution of the users' positions to make it clearer. The new sentence is reproduced below.

“Moreover, we assume that the positioning processes take place periodically. In each positioning process, the positions of the users are assumed to be independent and identically distributed random variables, each following a distribution Γ^U , i.e., $\mathbf{p} \sim \Gamma^U$.”

In Sec. IV-B2, we have rewritten the second paragraph, where we have improved the explanation for the detailed working principle of the fine-tuning technique in the proposed algorithm. The rewritten sentences are reproduced below.

“We note that the feature extractor optimized for $\mathcal{T}(\mathcal{D}_{\text{src}})$ is also effective for $\mathcal{T}(\mathcal{D}_{\text{tar}})$ as the channels in both domains follow the same structure, i.e., (1)-(9), and the same DA beamforming configurations are employed. Therefore, the method for feature extraction needs little adaptation, and $\mathbf{w}_{\text{feat}}^{(t)}$ can be frozen or adapted with a very low rate η_{feat} . In contrast, the regressor has to be adapted substantially to handle the differences in the extracted feature values caused by the different environmental characteristics. Thus, we adapt the regressor with a large learning rate denoted by η_{reg} . Though the amount of local user data collected in \mathcal{D}_{tar} is small, the adaptation of the regressor can still be done effectively since the regressor only contains the output layer of the MLP with a small number of trainable parameters.”

In Sec. V-A, we have rewritten the description of the location of the anchors and the criterion for users to be considered to be near an anchor. The rewritten sentences are reproduced below.

“Each data-label pair comprises a received signal matrix and the corresponding position label. A user obtains a position label when it is within 0.25 m of one of 10 anchors.

For simulation, the positions of the 10 anchors are drawn from a uniform distribution within the ROI.”

Moreover, in the description of Fig. 7 in Sec. V-B, we have revised a sentence to clarify the target performance criterion we considered in the comparison. The relevant text is reproduced below.

“Moreover, we calculate the average relative gain in terms of the required number of epochs for the MSE of positioning to drop from its initial value to below 0.5 m^2 . The corresponding values are provided in the sub-headings of Figs. 7(a)-(f), e.g., $\downarrow 37.3\%$ in Fig. 7(a).”

B. Some issues are not clear to me, the presented algorithms and results seems depends on an extra infrastructure for the positioning labels, some beacon is required to estimate the position of the users?

Response: We thank the reviewer for this comment. In this paper, as described in Sec. II-C3, we assume that a user can get a position label when it is near a few “beacons” in the region of interest (ROI), which we refer to as *anchors*. It is worth noticing that this assumption does not require extra infrastructure to be deployed or eliminate the benefits of our proposal. This can be justified as follows.

First, this assumption can be satisfied by already existing infrastructure; therefore it does not require extra infrastructure to be deployed. Specifically, when a user conducts near-field communication with the surrounding infrastructure (e.g., smart gate machines, point-of-sale machines, and RFID tags), the infrastructure can be considered as anchor, and a position label can be obtained since the position of the infrastructure is known and the user is in its proximity [37]. Infrastructures that support such short-range positioning have already been widely deployed in practice because of the prevalence of the Internet of Things (IoT) [38].

Second, short-range positioning cannot position a user within a large ROI as HoloFed does; therefore it does not eliminate the benefits of our proposed approach. In general, existing infrastructure can only provide a small number of anchors that are not densely deployed, and thus such short-range positioning capability cannot provide consistent high-precision positioning services to users within a large ROI. In comparison, our proposed HoloFed can provide high-precision positioning services within a large ROI, and thus the assumption that users can obtain position labels in the proximity of a small number of anchors does not degrade the benefits of

our proposed approach. Actually, one of the merits of our proposed HoloFed is that HoloFed can effectively merge the information provided by RHS-based signal transmission with the limited labels provided by existing short-range positioning techniques to achieve high-precision large-range positioning with environmental adaptability.

To clarify this in the manuscript, we have added a footnote in the second paragraph of Sec. II-C3, where we justify that short-range positioning techniques are supported by the existing and widely deployed IoT infrastructures. The added footnote is reproduced below.

“⁶Here, an anchor refers to a location where the users can obtain their position labels based on short-range positioning techniques. Such short-range positioning techniques can be readily supported by near-field communication (NFC) of the users with the existing Internet of Things (IoT) infrastructure [37], [38].”

Besides, we have added an explanation in Sec. II-C3 to clarify that with the help of the position labels collected near a small number of anchors, HoloFed can achieve high-precision large-range positioning with environmental adaptability. The added part is reproduced below.

“Exploiting the local gradient of the positioning loss w.r.t. the position labels collected near a small number of anchors, HoloFed adapts its position estimator to the actual deployment environment for achieving large-range positioning with high precision.”

C. The novelty of this manuscript is in doubt due to the paper accepted in ICC.

Response: We thank the reviewer for this comment. To address this comment, we note that our manuscript has four major novel contributions compared to our conference paper accepted for presentation at IEEE ICC [1] (arXiv:2303.15686).

Firstly, in this paper, we have proposed to exploit the federated learning technique to achieve environmental adaptivity for RHS-based ISAC systems while preserving user position privacy. To be specific, we have designed a new federated positioning protocol in Sec. II-C to coordinate the BS, RHS, and users, where the users can help the adaptation of a shared position estimator in a federated and privacy-preserving manner by uploading only the noisy gradient information of their local positioning losses. Based on the newly designed protocol, we have formulated a positioning error minimization problem that is essentially different from the problem considered in our ICC paper, as the problem in this paper involves a federated learning process.

Secondly, to solve the digital and analog (DA) beamforming optimization sub-problem, i.e., (SP1'), more efficiently, we have proposed a new proximal stochastic descent algorithm with alternating DA beamforming updates. Compared to the algorithm proposed for DA beamforming

optimization in our ICC paper, the new algorithm can handle a much larger number of sample points within a given time. Specifically, thanks to the newly designed algorithm, we could extend the number of sample points from $N_{\text{sam}} = 1200$ in our ICC paper to $N_{\text{sam}} = 10000$ in this paper. By this means, the sample points can represent the region of interest (ROI) more comprehensively, which is also beneficial to the positioning precision of HoloFed after environmental adaptation.

Thirdly, to handle the federated adaptation of the position estimator, we have designed a new algorithm based on the fine-tuning technique in transfer learning in Sec. IV-B. This algorithm is essential for providing the considered system with environmental adaptivity because it allows the position estimator to be pre-trained in a source domain and adapted to a different target domain by using only gradient information of the limited users' local datasets.

Fourthly, we have derived a novel proposition, i.e., Proposition 3, for the convergence rate of the federated adaptation in the case that users upload noisy local gradients. Based on this proposition, we have proposed a new user scheduling algorithm for selection of the user for local gradient upload in order to optimize the convergence rate and uploading efficiency. Based on the simulation result in Fig. 7, the proposed user scheduling algorithm achieves a 11% ~ 37% faster convergence rate compared to the benchmark.

In summary, our JSAC manuscript has been substantially revised and makes four distinct novel contributions compared to our ICC paper [1]. The ICC paper contains only a subset of this manuscript, where only the multi-band RHS-based ISAC system model and the CRLB for positioning precision parts are reported. Thus, we believe our manuscript meets the high technical standards maintained by JSAC and is novel work having its own merits.

To clarify the novel contributions made by our JSAC paper, we have added a corresponding statement following our main contributions listed in the introduction, which is reproduced below.

“Compared to its conference version [1], this paper proposes the application of FL for achieving privacy-preserving environmental adaptivity. Furthermore, it provides new optimization algorithms, which enhance the efficiency of the DA beamforming optimization by proximal stochastic descent, handle the insufficiency of users' local data by transfer learning, and improve the efficiency of user scheduling in FL based on an new analytical result of the convergency rate.”

D. Must be explained the novelty and exact improvement respect to the paper accepted in ICC, it seems to be derivative and this should be strongly considered in JSAC.

Response: We thank the reviewer for this comment. As we have explained in detail in the response to Comment C, our JSAC manuscript makes four distinct novel contributions compared to our IEEE ICC 2023 paper [1] (arXiv:2303.15686):

- We propose to exploit the federated learning technique to achieve environmental adaptivity in a privacy-preserving manner and propose a federated positioning protocol for this. In contrast, the ICC paper has not considered environmental adaptivity.
- We propose a new proximal stochastic descent algorithm with alternating DA beamforming updates, which enables the proposed system to handle more sample points in the ROI efficiently compared to the algorithm reported in our ICC paper.
- We propose a new algorithm based on transfer learning to enable users to adapt the positioning estimator effectively. Our ICC paper does not contain such an algorithm since it does not involve a process for users to train the positioning estimator with federated adaptation.
- We derive a novel proposition that analyzes the convergence rate of the federated adaptation in the case that users upload noisy local gradients. Based on this proposition, we propose a new user scheduling algorithm to optimize the convergence rate and uploading efficiency.

Considering the above four distinct novel contributions, our JSAC manuscript is not a derivative work of our ICC paper. The JSAC paper introduces novel and significant improvements for MB-RHS-based ISAC systems to enable environmental adaptivity, user position privacy, and enhanced system efficiency in terms of DA beamforming optimization and training of the position estimator. Therefore, we believe our manuscript meets the high technical standards of JSAC.

RESPONSE TO REVIEW 3

A. *As this work has been accepted in part for presentation at ICC, the authors should explain in the introduction how this work differs from the work in their conference paper explicitly.*

Response: We thank the reviewer for this comment. To address this comment, we note that our manuscript has four major novel contributions compared to our conference paper accepted for presentation at IEEE ICC [1] (arXiv:2303.15686).

- We propose to exploit the federated learning technique to achieve environmental adaptivity in a privacy-preserving manner and propose a federated positioning protocol for this. In contrast, the ICC paper has not considered environmental adaptivity.
- We propose a new proximal stochastic descent algorithm with alternating DA beamforming updates, which enables the proposed system to handle more sample points in the ROI efficiently compared to the algorithm reported in our ICC paper.
- We propose a new algorithm based on transfer learning to enable users to adapt the positioning estimator effectively. Our ICC paper does not contain such an algorithm since it does not involve a process for users to train the positioning estimator with federated adaptation.
- We derive a novel proposition that analyzes the convergence rate of the federated adaptation in the case that users upload noisy local gradients. Based on this proposition, we propose a new user scheduling algorithm to optimize the convergence rate and uploading efficiency.

In summary, our JSAC manuscript has been substantially revised and makes four distinct novel contributions compared to our ICC paper [1]. The ICC paper contains only a subset of this manuscript, where only the multi-band RHS-based ISAC system model and the CRLB for positioning precision parts are reported.

To clarify the novel contributions made by our JSAC paper, we have added a corresponding statement following our main contributions listed in the introduction, which is reproduced below.

“Compared to its conference version [1], this paper proposes the application of FL for achieving privacy-preserving environmental adaptivity. Furthermore, it provides new optimization algorithms, which enhance the efficiency of the DA beamforming optimization by proximal stochastic descent, handle the insufficiency of users’ local data by transfer learning, and improve the efficiency of user scheduling in FL based on an new analytical result of the convergency rate.”

B. In the introduction, the authors have considered using the multi-band RHSs, which are a relatively new RHS design, but in the introduction and the system model, they have not provided sufficient supports for the multi-band RHSs to be physically practical. The authors should provide more information on the existing multi-band RHSs or the potential implementation methods to realize them.

Response: We thank the reviewer for this comment. In the introduction, we have provided a reference, i.e., [18], to support the feasibility of multi-band (MB) RHSs. Specifically, in [18], the authors have shown in [18, Fig. 8] that their proposed metamaterial surface, which is equivalent to an RHS in our paper, can operate in multiple frequency bands at 9.0 GHz, 9.5 GHz, 10.5 GHz, and 11.0 GHz, while different beamforming configurations need to be used in different bands. This existing design supports the feasibility of the MB-RHS considered in our paper.

In addition to [18], we have also found support for MB reconfigurable intelligent surfaces (RISs) in several recent works. As RISs and RHSs are both composed of metamaterial antenna elements, the success of the implementation of MB-RISs also supports the feasibility of MB-RHSs. In [23], the authors designed and implemented a dual-band RIS that can operate at 2.4 and 5.8 GHz. In [24], the authors have designed and implemented a dual-band metasurface that operates in the bands at 6 GHz and 9.8 GHz. The design and implementation methods for metamaterial elements reported in these works can also be adopted for the implementation of MB-RHSs.

To clarify this issue in the manuscript, we have revised the fifth paragraph of the introduction to emphasize that the feasibility of MB-RHSs has been validated in existing references. The revised sentences are reproduced below.

“The feasibility of MB-RHSs has been verified in [18], where an MB-RHS capable of operating in bands at 9.5, 10, 10.5, and 11 GHz is realized. Moreover, in [23], [24], the authors prototyped RISs employing meta-elements capable of operating in two bands.”

C. In Section II-C1, the authors assumed the OFDM transmissions in multiple bands are performed sequentially, which may incur extra overheads. The reason for the transmissions in multiple bands not being conducted simultaneously needs to be clarified.

Response: We thank the reviewer for this comment. In this paper, the main reason for why the transmissions in multiple bands are performed sequentially is to make our proposed protocol compatible with the practical limitations of MB-RHS designs and to reduce the hardware requirements

on the users and the base station (BS). Specifically, due to the frequency selectivity of meta-elements [18], [19], their radiation coefficients for a single configuration may differ significantly in bands, i.e., the configuration yielding the most effective beamforming pattern in one band may lead to undesirable beamforming patterns in other bands. By assuming sequential transmissions in different bands, the MB-RHS can control the configurations in different bands independently, so that in each band the best beamforming pattern can be employed. This assumption is in accordance with the design of the RHS in [18], where the RHS operates in multiple frequency bands at 9.0 GHz, 9.5 GHz, 10.5 GHz, and 11.0 GHz, while different beamforming configurations need to be used in different bands.

Besides, if the OFDM transmissions in multiple bands are conducted in parallel, the BS and the users have to possess the capability to transmit and receive signals in multiple bands in parallel. This is a demanding requirement for the hardware, as the BS and the users need to have RF chains supporting ultra-wideband signal transmission and reception.

To clarify this point in the manuscript, we have revised the first paragraph of Sec. II-C1 to explain the reason for assuming sequential transmissions. The corresponding text is reproduced below.

“Thus, a state configuration that creates a desired beamforming pattern in one band may lead to undesired beam patterns in other bands. Therefore, to design favorable beam patterns in all bands, we assume that the OFDM transmissions in the N_B bands are performed sequentially, allowing the states of the meta-elements to be configured independently in each band. This approach also reduces the hardware requirements for the BS and the users as their RF chains do not have to support ultra-wideband signal transmission and reception.”

Nevertheless, it is worth noticing that if the RHS can control the beam patterns in multiple bands independently and the BS and users support parallel transmission and reception in multiple bands, then HoloFed can be easily extended to this case. To clarify this in the manuscript, we have added a footnote in the first paragraph of Sec. II-C1, which is reproduced below.

“⁵If the RHS can independently control the beam patterns in multiple bands, HoloFed can be modified to account for parallel transmissions in these bands, assuming the hardware of both the BS and users is capable of supporting it.”

D. In Section II-C3, what is the time complexity of the MMT phase?

Response: We thank the reviewer for this comment. Based on Fig. 3, the MMT phase has linear time complexity with respect to the number of bands, i.e., N_B , and the number of frames transmitted in each band, i.e., F . Besides, since the BS broadcasts the frames to all the users at the same time, the MMT phase has constant time complexity with respect to the number of users. Consequently, its time complexity is given by $\mathcal{O}(N_B F)$.

To clarify this point in the manuscript, we have added an analysis on the time complexity of the MMT phase in Sec. II-C1, which is reproduced below.

“ Based on Fig. 3, the MMT phase has linear time complexity with respect to (w.r.t.) the number of bands and the number of frames transmitted in each band. Besides, since the BS broadcasts the frames to all users at the same time, the MMT phase has constant time complexity w.r.t. the number of users. Consequently, its time complexity is given by $\mathcal{O}(N_B F)$.”

E. In Section IV-A1, it is not very clear how using multiple bands benefits positioning. The authors may want to explain this in detail.

Response: We thank the reviewer for this comment. The benefit of using multiple bands for positioning is two-fold. *Firstly*, based on (16), (17), it can be observed that the CRLB for positioning is inversely proportional to the summed Fisher information of multiple bands. Therefore, if more bands are jointly used for positioning, the Fisher information in the received signals increases, leading to a lower CRLB for positioning and thus a higher positioning precision.

Secondly, using multiple bands is more beneficial for positioning than using a single band with larger bandwidth or more frames. This is due to that the correlation between the received signals in different bands is much lower than that in the same band based on (7) and (8), which results in a higher Fisher information contained in the received signals. This can be demonstrated by the following example: Suppose we estimate a 1D position p based on received signals $y_1 = (h^{\text{LoS}}(p) + h_1^{\text{MP}})x$ and $y_2 = (h^{\text{LoS}}(p) + h_2^{\text{MP}})x$, where $h^{\text{LoS}}(p)$ is the LoS channel gain w.r.t. p , h_1^{MP} and h_2^{MP} are random multi-path gains following the normal distribution, and x is the transmitted signal. If y_1 and y_2 are received in different bands, h_1^{MP} and h_2^{MP} will have close-to-zero correlation; otherwise, if y_1 and y_2 are received in adjacent frames or sub-bands, the correlation between h_1^{MP} and h_2^{MP} will be higher, denoted by $\rho > 0$. Based on [41, Eq.

(3.31)], the Fisher information of y_1 and y_2 w.r.t. p is proportional to $(1 + \rho)^{-1}$. Therefore, signals received in different bands are expected to lead to a lower CRLB.

To address this clearly in the manuscript while keeping the presentation concise, we have added a remark in Sec. IV-A1, which is reproduced below.

“ **Remark 4:** According to Proposition 1, the benefits of using multiple bands for positioning are two-fold. *Firstly*, the Fisher information from multiple bands adds up, leading to a lower value for the CRLB. *Secondly*, the fact that signals received in different bands are generally less correlated (due to less correlated multipath gains) also contributes to a lower CRLB. This can be shown by deriving the Fisher information of two correlated signals based on [41, Eq. (3.31)].”

F. In Section IV-A2, the reason why the proposed Algorithm 1 contains an outer loop and an inner loop needs to be further explained.

Response: We thank the reviewer for this comment. In the proposed algorithm, based on [43], the outer loop and the inner loop are used to enhance the efficiency of estimating the gradient of the objective function w.r.t. the optimization variables. Specifically, in the outer loop, the gradient of the objective function w.r.t. the optimization variables is initially estimated for a set of randomly sampled points in the ROI. Then, within each subsequent inner loop, the gradient estimate is further refined based on the calculated gradient deviation during the update of the optimization variables. By this means, the efficiency and precision of the gradient estimation can be enhanced, leading to an accelerated convergence rate.

To clarify this point in the manuscript, we have revised the second paragraph of Sec. IV-A2 to explain the reason for using the outer loop and inner loop. The corresponding text is reproduced below.

“In the outer loop, the gradient of the objective function w.r.t. the optimization variables is coarsely estimated for a set of sampling points in the ROI. Then, in the inner loop, the gradient estimate is iteratively refined based on the gradient deviation determined during the update of the optimization variables. This method enhances the precision of gradient estimation for limited sampling points, leading to an accelerated convergence rate [43].”

RESPONSE TO REVIEW 4

A. The work propose a system using RHS, to tackle the disadvantage of user position estimation for a diverse environment. Through this paper a first try to minimize the MSE using different algorithms is applied. Is a well written paper and understandable although is a little bit complicated to its way of organized. Your proposed algorithm seems to perform better according to the simulation results which are provided in the last chapter. Moreover the reviewer could not find any major flaw as far as any grammatical or spelling errors.

Response: We thank the reviewer for the positive feedback for our manuscript. We have addressed all of the reviewer's comments, which has significantly enhanced the quality and clarity of our paper.

B. For the time being it seems a fair good work for finding the exact position of the users through the proposed algorithm. I would like a clarification as to what impact the presence of interference between the users can have. Will the interference affect the performance of the aforementioned, since all the metrics which will be used will be affected from the interference?

Response: We thank the reviewer for this comment. Two different types of interference between the users may be considered as follows.

First, in the positioning process, users do not suffer from active user interference caused by the active signal transmissions of other users. This is because, as described in Sec. II-C1, in the multi-band multi-pattern transmission (MMT) phase, the users only passively receive the signals transmitted by the BS and do not transmit actively. Although the users need to actively transmit their local updates to the BS in the federated adaptation phase, they are scheduled by the BS so that only one user transmits at a time.

Second, in the positioning process, users receive signals that were passively scattered by the bodies of other users, which constitutes passive interference between users. As the reviewer has mentioned, such passive user interference affects the observations used for positioning, i.e., the received signals, and thus affects the performance of HoloFed. Since the scattering paths can be considered as random multipath channels, passive user interference can be accounted for by the random multipath channel gains that we have modeled in Sec. II-B3. Specifically, in (6), we model the covariance of the multipath channel gains of the received signals in band i by \mathbf{V}_i . If the passive user interference becomes larger due to, e.g., a larger number of users in the system,

the powers of the multipath channel gains will become larger, leading to a magnified covariance matrix \mathbf{V}_i . Then, based on (6) and Proposition 1, if \mathbf{V}_i is magnified by a factor of $X \in \mathbb{R}$, the Fisher information will decrease approximately X times, leading to an X -fold increase in the CRLB.

To clarify this, we explain in the revised paper that passive interference can be accounted for by covariance matrix \mathbf{V}_i . The added explanation is reproduced below.

“We note that \mathbf{V}_i can also account for the passive interference among users, i.e., the interference caused to a given user by signals passively scattered by the bodies of other users; because the scattering paths can be modelled as random multipath components.”

Besides, we clarify that the users in HoloFed do not suffer from active user interference by adding a remark, namely Remark 2, at the end of Sec. II-C. The added text is reproduced below.

“**Remark 2:** In HoloFed, users do not suffer from active user interference caused by signal transmissions of other users because they only receive signals in the MMT phase and are scheduled to transmit their local updates one at a time in the federated adaptation phase.”

Moreover, we have added a remark, namely Remark 3, following Proposition 1 to clarify the impact of passive user interference, which is reproduced below.

“**Remark 3:** Based on (6) and Proposition 1, the impact of passive user interference on the CRLB can be analyzed: If the magnitudes of all entries of \mathbf{V}_i increase by a factor of X times due to larger passive user interference ($\forall i \in \{1, \dots, N_B\}$), then based on (16)–(18), $\text{CRLB}(\mathbf{p})$ will increase approximately by a factor of X as well.”